The Proceedings of the Third International Conference on Developing Mathematical Resilience

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MATHEMATICAL RESILIENCE NETWORK

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Introduction to the Proceedings

The Third International Conference on Developing Mathematical Resilience features papers from researchers, academics and, importantly, from those who work with people of any age who are learning mathematics. Alongside people working in academia or working as consultants, in these proceedings you will find papers from teachers and others working with parents and learners in the classroom and in other learning environments.

Having mathematical resilience enables people to overcome the barriers that learning mathematics presents. Many papers in these proceedings deal with overcoming mathematics anxiety which is prevalent globally and helping learners know how to protect their well-being and still learn mathematics.

All the papers in these proceedings are research-based. They present research dealing with the affective domain in its broadest sense and how mathematical resilience is making an impact in learning environments from higher education to school and in homes, in countries across the world. The conference offered an innovative, inclusive and friendly environment for participants to showcase their research and emerging practice. These proceedings aim to do the same.

Keynote address 1

Mathematical Resilience – Where it came from and where it is going

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Abstract

In this paper, the origins of the construct of mathematical resilience are discussed and perhaps more importantly where the ideas around mathematical resilience are going. The background to the construct is presented along with the initial experiences we had when coming to understand the power of defining and shaping this construct which stood in opposition to the prevalence of mathematics anxiety. Self-determination theory is used to show how working to develop mathematical resilience meets learners needs' and allows them to take a positive stance towards mathematics and be motivated to learn.

Introduction

The construct that we call mathematical resilience grew out of conversations between Sue and Clare about mathematics anxiety and how that anxiety seemed to affect so many people at all levels of society. We explored practices prevalent in mathematics teaching that we are now certain contribute to anxiety and sought out pedagogical practices that would work against anxiety and develop a positive stance towards mathematics. We started to understand that stance as mathematical resilience.

Reading Ashcraft and Krause (2007) led to an understanding that mathematical anxiety was prevalent in society and can be a destructive phenomenon creating avoidance and that it is sometimes rooted in trauma that has been experienced in classrooms or indeed at home. Mathematics anxiety did not have to be the outcome of teaching mathematics, but it all too often was. We agreed with Nardi and Steward (2003) that mathematics teaching is often experienced as T.I.R.E.D: tedious, isolated, rote, elitist, and depersonalized. We were also aware that affect (emotions, attitudes, beliefs, values) was often ignored when teaching and researching mathematics. As a consequence, mathematics anxiety was often dismissed as insignificant and irrelevant by academics and its existence continues to be considered unimportant by some today. Worryingly Ofsted's recent research review into mathematics (Ofsted, 2021) does not mention the possibility of mathematics anxiety being developed and recommends that "lessons incorporate timed testing to help pupils learn maths facts to automaticity" (p.30) despite research that indicates such pedagogy can lead to anxiety.

Working together in the education department of Warwick University gave us time to explore what it meant to teach mathematics effectively. We read papers by and worked alongside colleagues such as John Mason, Candia Morgan, Barbara Jaworski and Malcolm Swan, who knew what effective mathematics teaching could look like. Alongside these experiences we saw acute anxiety about engaging with anything mathematical. We met a mother who felt she could not help her daughter with the mathematics work that was baffling her, as she was too afraid to do so. A highly successful colleague told us that he could not and would not engage with numbers and always got someone else to do any calculations for him. When Sue asked a group of people

who were training to teach primary school pupils how mathematics made them feel, 90% said it made them anxious. We started asking ourselves how we could work against this prevalent anxiety. Slowly and painstaking we built the ideas that constitute the construct of mathematical resilience.

We published in Mathematics Teaching in order to begin to gather our thoughts (Lee and Johnston-Wilder, 2010). These ideas were taken up by Janice Kooken who investigated them statistically. She found a cohort of students training to be actuaries, who might be expected to be resilient mathematically and contrasted them with the general student population at her American college. The results (Kooken; Welsh; McCoach; Johnston-Wilder and Lee, 2013) show that the facets of struggle, value and growth mindset are viewed differently by the two populations, leading us to believe we were developing something that made sense.

We offered to work with the staff and pupils of schools to build mathematical resilience and to present the potential for greater success in examinations. We tried out several ideas at this time; one was working with a school to set up a dramatic (imaginary) robbery where each group of students was tasked with solving the crime by using mathematics to support or discount the alibis offered. We learned much from this project, in particular that the students would happily join in and would engage in the mathematical calculations, but they needed support to do so. We also learned that some staff were reluctant to engage in unprepared mathematics of the sort required in this situation. Some were happy to ask 'what if …'? questions to get groups started e.g. what if we thought about speed and distance? and to ask for and challenge assumptions, others would not engage, which discouraged the students. This is now unsurprising; in a performativity culture staff cannot afford to suggest something they do not know for certain will work. Performativity demands that mathematics teachers do not make mistakes. Further, if staff have a fixed mindset themselves and therefore need to affirm their own intelligence through 'getting things right', they will not engage as making mistakes is perceived as too risky. This affirmed the importance of everyone, teachers and other adults included, developing a growth mindset (Dweck, 2000).

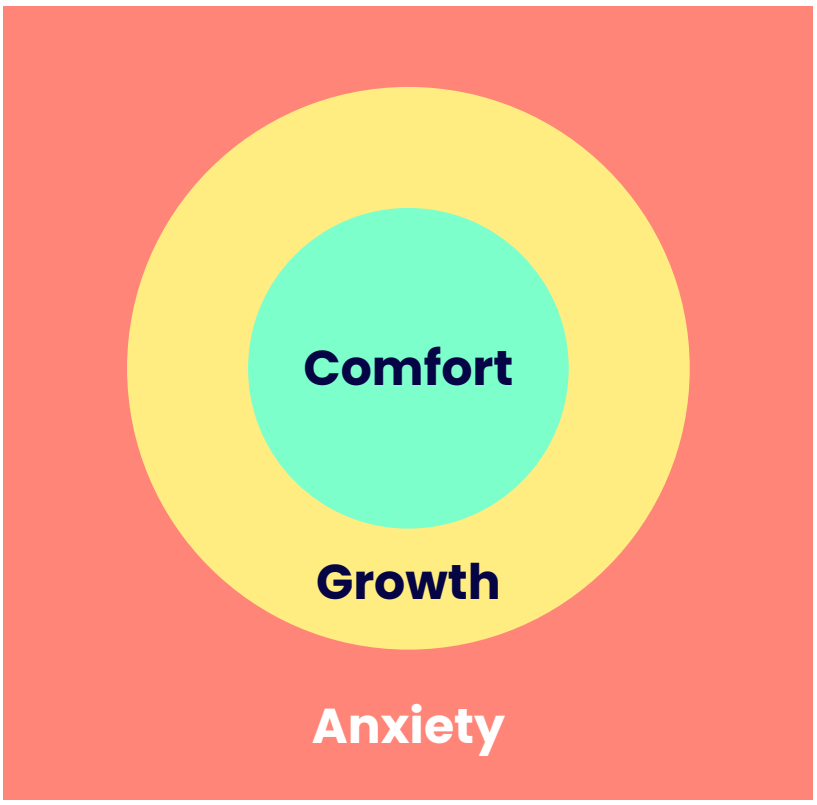
The facets of mathematical resilience were beginning to become apparent. Support must be part of an ethos that develops mathematical resilience. Where the right kind of support was offered when needed, mathematical learning would grow. We were invited to help a school offer support to its year 11 pupils. It was vital that the school improved its GCSE results, but they struggled to recruit mathematics teachers. In discussion, the school decided that offering their pupils support with their mathematics from a wide group of adults would increase the support on offer and help decrease the idea that only an elite can do mathematics. The idea of "Maths Angels" was born. Any adult in the school could be a "Maths Angel"; the school receptionist and bursar volunteered as well as teachers from many different subjects. A meeting was convened where the agenda was how to offer support with mathematics. Our original principal was that these adults would not need any more mathematics to provide support, but they insisted. The attendees were given some mathematics to work on with the idea of offering support to one another. By examining the support they needed, we hoped they would decide on how support should be offered to anyone requesting help from them. We did not get very far. A Design and Technology teacher took one look at the problems offered, burst into tears, and left the room. The discussions that followed made us see just how difficult most adults find offering support, even when they think they might be able to. Their own fears and need for support must first be addressed before they can support others.

So, we had growth mindset and the need for the right kind of support as pillars on which developing mathematical resilience could be built. However, as yet we had not taken into account the views of the important people in all our endeavours, those of the learners. That is where our attention was focused next. The results of this piece of research were published as Lee and Johnston-Wilder (2013). We took the view that the students would think that there was only one way to learn mathematics, the one that they were regularly exposed to, so before we asked them about their preferred learning environment, we exposed them to different ways of learning. This had the effect of enabling us to collect data on whether the ideas we had put together

about a learning environment which fostered mathematical resilience was recognised and appreciated by learners themselves as well as allowing the learners voices to be heard. From this we learned that in the right environment learners would challenge themselves, seek the support they needed and were prepared to struggle to understand the mysteries of mathematics. Their resilience would see them through if they were set free from the shackles of an anxiety-causing environment. They wanted to be challenged with work that extended their ideas and to support one another, to be the 'more able peer' (Vygotsky, 1978) for one another. They did not want to compete with one another or be 'shown up' because they were not quick enough to understand, rather they wanted time to think, reason and understand. They had no interest in remembering procedures, it was meaning and understanding that they sought.

That gave us the four facets of mathematical resilience, a growth theory of learning, a feeling of being valued in the community of those learning mathematics and of mathematics being of personal value, a comprehension that learning mathematics requires struggle and will be challenging and an understanding of how to recruit the support needed to meet those challenges. Now we went in several directions, led by our individual contexts. Clare began work with the Open University's PGCE and interacted with schoolteachers at both primary and secondary level, building an understanding of ways of teaching that build resilience in mathematics. Sue worked first developing coaches to support learners re-taking GCSE mathematics, often for the third or fourth time, and then developed a coaching course for those charged with supporting learners of mathematics who were themselves anxious. Sue's work made clear that when the damage had been done and mathematics anxiety was already disabling, any engagement with mathematics was difficult. However, learning about the ideas of coaching alongside an explanation of the likelihood of debilitating anxiety being present, can help the putative coach recognise and overcome their own anxiety allowing them to support mathematical learning. She established that working with a coach can help learners overcome their anxiety and start the learning process in a way that a more traditional teacher would find difficult. The disabling effects of fixed mindset, elitism and an over-regard for speed as a marker of mathematical thinking were often so engrained that spending time working on mathematical ideas was almost impossible for many learners. They needed someone alongside them helping them slowly move towards mathematical learning, not ahead of them beckoning them forward.

As we discussed our work with schools and teachers, we developed the growth zone model to explain the ideas and myths about learning mathematics which had built up over the many years that people have been required to learn mathematics when at school. The comfort zone was designed to help dispel the myth that rows and rows of "ticks" showed good mathematical learning. It was often a revelation to learners that challenge was necessary if learning was to happen but with challenge came uncomfortable feelings, feelings of risk and insecurity, as they came to understand more. Moving into the growth zone would mean feeling some 'growing pains' but with the support of an environment that



develops mathematical resilience they would be able to enter their personal growth zone and slowly learn to stay there longer. Many learners told us about the narrowness of their growth zone, they would move into it and then almost immediately out of it into the red zone, where anxiety prevents learning. We recommended deep calming breathing when this happened, taking the time they needed and not feeling rushed but to calmly try again when they could. Most of all we advocated seeking the support that worked for them prior to trying to accept the challenge again. Understanding that anxiety was prevalent, it was not just them, was vital, as was knowing their anxiety had been caused by ways of teaching mathematics which were not the only ways of teaching. We began to define mathematical resilience as what is needed to thrive in the growth zone.

That students' mathematical learning thrives, along with their self-esteem and self-efficacy in environments designed to build mathematical resilience has been proven over and over in many learning situations, many of which are discussed in these proceedings. Changes in the learning environments can be huge when facilitators of learning come to understand the unintended consequences of some ways of teaching but more often they are small, as the ideas fit naturally with the pedagogies of teachers whose purpose in taking up the profession is to nurture and empower the learners with whom they interact, see for example Lee & Ward-Penny (2022)

Deci and Ryan's (1985) self-determination theories began to allow us to see why so many people talked in terms of the damage that had been done to them personally, the shame and frustration they felt when their need for understanding was met with the prevalent fixed mindset view that some people just cannot learn. Self-determination allows people to feel they have control and can make choices and manage their own life; without this their psychological health and well-being is compromised. It impacts motivation, as people feel motivated when they feel their own actions will affect the outcome in a context. Deci and Ryan (1985) suggest that having high self-determination fosters success. Self-determination theory suggests three innate and universal psychological needs must be met if people are to have the autonomous motivation to grow and change, an important aspect of learning. Self-determination results from the need for competence, connection, and autonomy being fulfilled. We found we could relate each of these needs to the main factors we had identified as necessary to build mathematical resilience:

Autonomy: this is the need for people to feel in control of their own behaviours and goals. As we researched, we recognised that when talking about learning mathematics people often felt out of control. An example of a mathematically resilient pedagogy is that people must feel supported but feel supported in ways that they choose to help them reach their own goal, which is often understanding and connecting ideas, not simply remembering a procedure. Ideas around the personal value of mathematics are also linked to the need for autonomy and the notion of autonomous motivation.

Competence: this need indicates that when people feel that they have the skills needed for success, they are more likely to take the actions that will help them achieve their goals, in this case to engage in the struggle to learn mathematics. Pedagogies that emphasise growth mindset stress that competence is not the 'you have it, or you don't' aptitude associated with the prevalent fixed mindset, but rather that capabilities grow as they are worked on with the support that is needed at the time. This also relates to the mathematically resilient idea that it is important to understand how to work at mathematics. Mathematical learning requires struggle because as the head of Ofsted, Amanda Spielman (2021) says "maths is mysterious and difficult" and learners need to "build confidence in their ability". However, engaging in that struggle should not be a lonely endeavour, collaboration and community are also part of a mathematically resilient environment.

Learners also need to build competence in self-safeguarding. Building such competence enables learners to carry on learning mathematics in contexts that they would previously avoid or in which they would shut down. Self-safeguarding (Mackrell & Johnston-Wilder, 2020) is the term used to denote the learner's awareness of how they are responding in a given context, understanding of their perception of the context as a threat and active choice of a response that will enable them to deal with the threat more productively.

Relatedness: meeting this need will mean that learners experience a sense of belonging and attachment to other people. Therefore, an environment that allows mathematical resilience to grow is one where collaboration is the norm, although individual time is an allowed autonomous choice. The building of a community that works together to maximise mathematical learning in all its members is an important part of growing mathematical resilience.

Inclusion is a vital aspect of working with self-determination theory, and being part of a community that learns and works together is about helping its members learn what does and does not work for them as individuals. Where learning environments are tightly controlled by a leader of learning that insists on one way of proceeding, very few of the actors within that environment will have their needs met. They will not become self-determined and their mental health and well-being are likely to be undermined. An environment that builds emotional awareness and regulation is needed because the 'red zone' is a constant threat when learning mathematics due to its nature. Therefore, teaching Benson's (2000) relaxation response and notions of "micro-mindfulness" are also important for developing mathematical resilience.

The next idea we want to cover in this article is "where next?" We know that meeting learners' need for self-determination will result in people who are willing and able to control mathematical ideas and to think and reason mathematically in ways that are likely to allow society to thrive and more people to flourish within society. We also know that we are not alone in developing these ideas and it is in linking beyond mathematical learning that answers the "where next?" question. We are currently linking mathematics anxiety with notions of trauma-informed teaching practices (see for example Jennings, 2018), since mathematics anxiety is clearly a trauma to overcome. We also understand that compassion is an underused but vitally important part of teaching that can develop a fairer society more able to cope with the complex issues that will affect today's learners in the future (Warwick, 2016) and which mathematics will be an important tool in managing.

The growth zone model has been a very powerful visual stimulus helping people understand the gamut of emotions that can and usually do accompany learning mathematics. It has links with the discomfort zone (NHSGGC, 2020) which is used widely in the NHS when discussing learning as well as pain management. There are also echoes of the growth zone model in Brown's (2022) work on creating brave spaces in learning environments where discussion and therefore learning can move beyond the safe and into the challenging and where risk can be accepted. In these links it can be seen we are exploring psychological safety and how learners can safeguard their vulnerable selves and keep on learning when they move beyond a mathematically resilient classroom.

So often we see that establishing a mathematically resilient learning environment allows people to heal and flourish. We hear comments such as:

The mums were so relieved to hear that maths anxiety is "real" and that they weren't the only ones who felt physically sick before maths lessons and made to feel stupid.

They loved the approach of safeguarding and hadn't previously thought about healing the maths trauma they had carried for decades.

Everyone needs to hear this vital message.

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Keynote address 2

Resilient teacher activity that promotes deep mathematical understandings

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Abstract

An argument for increasing teacher resilience to reduce the need to address student anxiety arising from absence of mathematical resilience is made. Theory and illustrating are employed to link general teacher resilience and teacher pedagogical problem solving during implementing of mathematical problem solving. Indicators of teacher resilience, and resilient actions that contributed to eliciting of students' thinking were identified from data collected from teacher interviews, and multiple camera capture of problem-solving activity and group worksheets in a Year 1 early elementary school class. Attention is drawn to the way a story book was adapted as part of problem-solving design to provide opportunities for 'big ideas' without raising teacher anxiety associated with difficult mathematics. Resilient actions and reflections from this teacher promoted student autonomy during group work and whole class sharing as mathematical constructs including the meaning of 'remainder' began to emerge. Links to mathematical resilience are made.

Keywords: mathematical problem solving; teacher resilience; enacted resilience, mathematics anxiety.

Introduction

Mathematical anxiety can be defined as "a set of physiological, cognitive, and behavioural reactions in situations where good performance in mathematics is expected" (dos Santos Carmo et al., 2019, p.403). It is prevalent in teacher-controlled mathematics classes (Lee & Johnston-Wilder, 2017). Changing from teacher-controlled mathematics learning environments to learning environments that enable autonomous student mathematical thinking has the potential to limit the development of mathematics anxiety (Csikszentmihalyi & Csikszentmihalyi, 1992) thus reduce the human resources required to support students in overcoming mathematics anxiety. This paper examines actions of a resilient teacher who implemented mathematical problem solving with opportunities for autonomous student thinking for the purpose of raising awareness of the role teacher resilience can play in reducing students' mathematics anxiety.

Theoretically framing this study

Resilience is a characteristic that enables a person "not merely to survive but to thrive under adverse circumstances" (Resnick, 2000, p.158), and to enact "academic, emotional, and social competence despite adversity and stress" (Nettles et al., 2000, p.47). Seligman's (1995) optimism is the resilience construct employed in this study. An optimistic problem solver sees not knowing how to proceed as temporary and able to be overcome through the personal effort of looking into the situation to identify what they could change to increase their chances of success. They can identify what is likely to work and what is unlikely to work and focus their energies productively.

They perceive their successes as permanent and take them on as characteristics of self. Research has linked resilience to student capacity to problem solve mathematically (Williams, 2014), and found that resilience can build over time as students make progress in developing mathematical insights (Csikszentmihalyi & Csikszentmihalyi, 1992; Seligman, 1995; Williams, 2011, 2014; Stillman et al., 2009). Developing mathematical insight is a creative activity that occurs during Flow (Csikszentmihalyi & Csikszentmihalyi, 1992). Flow is a state of high positive affect during creative activity. It occurs when a person or group autonomously set their own challenge that is almost out of reach, that requires the development of new skills (Csikszentmihalyi & Csikszentmihalyi, 1992) and / or mathematical concepts (Williams, 2002) to overcome it. Successes during flow can build resilience over time (Seligman, 1995).

Associations between resilience (optimism) and *pedagogical* problem solving have more recently become an area of research interest. By exploring ways in which resilience is enacted by teachers who have demonstrated the capacity to elicit creative mathematical thinking, this paper could raise awareness of the importance of this construct and the need to develop it in our teachers. It is argued that this can reduce the number of students developing mathematics anxiety during the learning of mathematics. The questions posed are: What types of indicators of optimism (resilience) have these teachers displayed? And in what ways do they enact resilience as they implement mathematical problem solving that promotes mathematical insights accompanied by high positive affect?

Research design

Context

This study is part of a broader study of optimistic characteristics of teachers, that support the design, implementation, and reflections about mathematical problem-solving activity that elicits mathematical understandings.

The research subjects are Earl and his Year 1 class in a small (5-teacher) rural elementary school in Australia. He was part of a professional learning program undertaken by the author, that extended across several years. Once he had observed the approach, Earl was immediately interested in the eliciting of deep mathematical thinking that occurred. He decided to focus on questioning to promote and sustain mathematical thinking: "so from the very first time that we spoke and we talked about maths … I could see this being something that would definitely help our students."

The Pedagogical Approach

The Engaged to Learn Approach (E2L) developed by the author over time as a teacher (see for example Williams, 2020) provides opportunities for flow conditions within which mathematical insights can develop. It is employed to develop informal mathematical understandings before a topic is formally taught. Consistent with Inoue et al. (2019), the development of the whole student rather than only their mathematical development is a focus. The approach involves groups of three to four students brainstorming ways to solve unfamiliar challenging problems. These groups were intended to be composed so they contained students with the same pace of thinking about new mathematical ideas (not the same level of performance). They give brief reports to the class each 10-15 minutes. This is followed by the teacher valuing as aspect of each report and posing questions (without hints or affirmations) that students are not obliged to pursue. This cycle of brainstorming, reporting, and teacher questioning is repeated several times. Two class rules designed to keep reporters emotionally safe are that class members cannot ask questions beyond the content reported and they cannot contradict ideas while the reporter is at the board. Such contradictions can occur during later reports if justification is provided. Class members can ask the reporter questions to gain a better understanding of what was presented. These constraints contribute to an environment which reduces judgements of the mathematical performances

of students, and increases social inclusion, thus reduces the likelihood of mathematics anxiety developing (dos Santos Carmo et al., 2019). Students feel emotionally safe taking risks as they explore and communicate their ideas.

The Task

For the E2L approach to work, the task needs to provide opportunities for students to discover and decide to explore mathematical complexities that were not apparent to them at the start of the task. To be accessible to many students, complexities that may become apparent should be able to be explored through multiple representations and various levels of mathematical sophistication. Those which are not accessible to some groups are not likely to be the ones they decide to explore. To increase the likelihood of the teacher perceiving the mathematics that could be elicited as accessible for them, the task was designed using a story book as stimuli. This carefully selected story book was also designed to stimulate student interest in the tasks. The story book 'The doorbell rang' (Hutchins, 1989) had large colourful pictures that would be easily seen by a class sitting on the floor around their teacher. The story as written was intended to *teach* mathematical ideas, which is not the intention of the type of task required in E2L. In the story as told in the book, the doorbell rings several times and each time the number of people needing cookies is a factor of 12. The task implemented by Earl in his Year 1 class commenced with the mother baking 12 cookies for her two children to distribute fairly amongst themselves. Turning this task into a 'conceptual task' from which students could develop new mathematical ideas was achieved by requiring groups (pairs or threes) to decide for themselves how many people would be at the door when the doorbell rings. Students then decide how many whole cookies each person gets to be fair.

Data collection techniques

Data selected for this analysis included that from multiple cameras in the classroom that captured the activity of nine groups (mainly pairs), the teacher or reporter addressing the class, and class members asking questions of reporters or responding to their teacher. Data from researcher (author) observation, group worksheets and the post-lesson video-stimulated conversation (interview) between Earl and the researcher was also included. This enabled study of teacher enacting of resilience in class and student outcomes influenced by this teacher activity [lesson videos], teacher and researcher identified classroom activity, teacher indicators of resilience [video stimulated teacher], and discussions of mathematical understandings of students evident on the video records [video stimulated teacher interview and student work sheets].

Analysis and results

Student groups used their worksheets to represent their problem arising from how many they decided were at the door. The number differed for each group, as did whether, or not, they remembered to include the two children inside in their count. The number of people different groups needed to allocate cookies to included 4, 6, 8, 12, and 17. The types of representations groups used on their worksheets differed from pictorial images of people and cookies to diagrammatic representations of circles with cookies inside them, numerical representations only (with no explanation), to both diagrammatic and numerical representations of the same problem with no links made between them. Groups were surprised by various complexities they encountered. These included different numbers of people to allocate cookies to when groups had the same number of people at the door (because some groups did not include the two children inside), finding they could not allocate all the cookies if only whole cookies were allocated, finding no-one could have any cookies if they were going to be fair and only use whole cookies (17 at the door), and being surprised to see different representations from different groups.

The pair allocating cookies to 8 people had a detailed pictorial representation of eight people (each with a cookie) on their worksheet and four small very faint dots in a bottom corner of the sheet. Earl knew, from discussion he had heard in their group, that this pair were aware that they had cookies 'left over'. Their reporter (Jenni, pseudonym) did not mention the extra cookies. She began to tell a story about the people on the worksheet: "we have eight people … she wants some cookies". Earl questioned, to elicit discussion of the remainders. Each time he varied his question, he acknowledged Jenni's story and drew attention back to the mathematics: "and what happened with the cookies when you had eight people at the door?" In his interview, Earl stated: "I was really hoping for her to uncover the 'left over bit' … I am conscious at the same time not to lead the thinking- and that's a really fine line". After Earl had asked several questions, Jenni acknowledged eight cookies were needed: "eight- I got twelve cookies". Earl invited class members to question. In his interview he stated this as a general strategy that he used: "… the questions … that the kids ask each other … often draw out [ideas I have not been able to elicit]". As Earl expected, this occurred. When a class member asked: "what's all those dots at the end" Jenni responded: "Cookies- more cookies". The focus was now on the extra cookies, so Earl asked a follow up question: "Oh- could you explain the 'more cookies' bit for us?". This did not elicit further information about these cookies. Having overheard their group discussions, Earl knew the pair knew more than they had communicated. He made the judgement that he and the class would not be able to elicit further at this stage. He had learnt from previous problem-solving activity that ideas that he and the students could not elicit at the time could emerge later. He made the following statement in his interview: " … I am putting that back more on the students with my questioning and seeing if they can- and *if they can't* do it- … I just leave it out there … *hoping* … another student … will be able to pick that up … [later]." Thus, Earl did not 'tell' but rather moved on to the next group.

In his interview, Earl reflected on his teaching practice: "My own teaching has completely changed from when I first started doing problem solving." He elaborated "… my questioning is something that I have improved in …" and elaborated on the effort involved in achieving this: "It's something that I have put a lot of thought into- it's not come over night- I've had to chip away at it …".

Without optimism / resilience Earl could not have elicited mathematical ideas

Earl saw *not knowing how* to elicit more information from the reporter as *temporary*. This is why he did not give up but rather altered his questions looking for ways to elicit more, and then invited class members to ask questions hoping to elicit even more. He has shown that he is able to look into the situation of not knowing how to proceed and identify what steps were likely to help with eliciting more. The personal effort he put into developing pedagogical problem-solving skills is evidenced in his descriptions of how he developed his questioning skills and his perception that his expertise is permanent. His descriptions of how he has changed as a teacher demonstrate that he has taken on this new teaching mantra as a characteristic of self.

Discussion and Conclusions

The role of teacher resilience in the implementing of mathematical problem-solving that elicits 'big ideas' through E2L has been illustrated. Resilience in the form of optimism (Seligman, 1995) was shown to contribute to teacher flexibility as they 'think on the run'. If a teacher does not perceive *not knowing* as *temporary*, they are not going to try to elicit new thinking if their first attempt does not work. If they are not able to *look into a situation* to identify what they could *vary* to increase their likelihood of *success*, teachers are not going to try different alternatives. Resilient teachers are aware of when their *personal effort* has contributed to *success*. They perceive the problem-solving expertise they have developed will be able to be used again (it is *permanent*) because it has become a characteristic of self (*pervasive*).

As success with creative mathematical problem solving builds resilience (in the form of optimism') in students (Csikszentmihalyi & Csikszentmihalyi, 1992; Seligman, 1995; Williams, 2002) and student resilience increases their capacity to problem solve (Williams, 2020), building teacher resilience for the purpose of increasing student resilience is crucial. The task and the E2L approach (Williams, 2020) also contributed to what was achieved. E2L develops the whole student (Inoue et al., 2019) rather than focusing only on their mathematical development, by providing an inclusive environment where students feel safe to take risks (dos Santos Carmo et al., 2019) and they learn to communicate clearly and question respectfully. Loving kindness is thus promoted (Baker & Johnston-Wilder, 2019). "The construct of mathematical resilience allows learners to manage and protect themselves from unhelpful emotions that may arise when mathematics becomes difficult to learn" (Lee & Johnston-Wilder, 2017, p. 270). Accessible tasks in an E2L environment should limit the developing of mathematical anxiety associated with mathematics being perceived as too difficult (Csikszentmihalyi & Csikszentmihalyi, 1992; Lee & Johnson-Wilder; 2017). Thus, these two constructs are mutually exclusive. Learning through E2L is not expected to include mathematics anxiety building situations. Further research is required to find ways to legitimise problem solving as a commonly used learning activity.

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Paper 1

Mentoring maths teachers through co-teaching to develop resilience: A case study in the FE sector

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Abstract

Enhancing students' experience in classrooms in the FE sector by improving instructional quality has been an ongoing challenge. Although mentoring is known to contribute considerably to developing mathematics teachers' competence and resilience, improving mentoring in FE deserves immediate attention. Using a case study, this research explores how mentoring can prove constructive, efficacious and transformational through co-teaching. A model of mentoring, Mentoring through Co-Teaching (MTCT), is investigated in order to further explore mentoring in the moment, to support teachers and mentors themselves become more skilled and resilient practitioners. Rich qualitative data, in the form of a semi-structured interview with the participant, observation reports, self-evaluation reports and learners' voice, are analysed to determine the impact of this study and recommendations are made to further improve teaching, learning and assessment in the FE sector.

Keywords: FE; further education; co-teaching; mentoring; resilience.

Introduction

Teaching is a complex and complicated activity. Teachers need some 'distinct skills' (Felder, 1993:176) and qualities in order to perform their duties and provide learners with the best possible learning experience. All teachers take years to develop expertise in teaching; become more resilient teachers (Day & Gu, 2013) and gritty individuals possessing not only passion but also perseverance for their long-term goals and to ultimately 'function at a professional level' (Felder, 1993:176). Mentoring can provide the repeated practice and feedback needed to speed this process. Observers traditionally watch the lesson and write a report highlighting strengths and areas for improvements. However, mentoring that takes the form of 'a presence, a being-with and co-teacher rather than a detached observer' (Roth, Masciotra & Boyd, 1999:780) seems more likely to support lasting improvements.

In this paper, I present a case study of a mentoring model (mentoring through co-teaching – MTCT) which seems positioned to help mathematics teachers become more resilient and also to help their students develop mathematical resilience. In this model, the mentor provides mentoring in the moment (Marciano et al., 2019) during the lesson and works as a facilitator to support the mentee. The mentor supports a teacher to transform their teaching and improve instructional quality.

Research Context and Case Study

As part of a pilot mentoring programme run by another Teaching, Learning and Assessment (TLA) Practitioner at my college, I had an opportunity to mentor one of my colleagues who teaches Functional Skills Maths in vocational areas. We started co-teaching and discovered

that mentoring through co-teaching helped us both further improve our individual skills and transformed us into more empowered and resilient professionals. During the co-taught lessons, I would discreetly step in when appropriate, demonstrate an aspect of TLA that I felt the mentee might need support with, and then step back. Through this mentoring programme, we built a good mentoring relationship with high levels of trust, confidence and respect. My mentee returned to the mentoring programme the following year because of the learning opportunities this programme offered. Although this case study involves only one participant, a great deal can be learned from the 'naturally occurring social situations' explored here 'in considerable depth' (Gomm et al., 2000) about how mentoring can prove constructive, efficacious and transformational through co-teaching.

Literature Review

Mentoring through co-teaching can empower mentees (Hobson, 2016) and enhance their instructional quality (Hattie, 2011) providing a rich learning experience. Mentoring through co-teaching has proved efficacious and constructive in a variety of contexts, for example: higher education; nursing; teacher education; and mathematics education. However, there seems to be no literature concerning mentoring as co-teaching to support FE teachers. It is the aim of this research paper to systematically focus on the insight, views and perceptions of mentoring through co-teaching in the FE sector. Mentoring is a one-to-one relationship between a relatively less experienced teacher and a relatively more experienced teacher aiming to 'effectively support the professional learning, development and well-being' (Hobson, 2016:1) of the mentee. In the current study, co-teaching is seen as the mentee assuming the responsibilities of the teacher whilst the mentor observes and intervenes during the delivery of a lesson as seems appropriate and helpful.

Tomlinson's Progressively Collaborative Teaching (PCT) (1995) approach gives a new teacher an opportunity to work with a more experienced teacher-mentor, in a non-judgemental fashion. The mentee progresses at their own pace to try out a wider range of more effective aspects of planning, preparation and teaching activity. Hellier & Davidson (2018) noted, 'team teaching can provide a rewarding educational experience for educators and students' (p.191) and 'team teaching as a form of mentorship becomes critical to new educator satisfaction and retention' (ibid). Finding the time for the recommended 'careful ongoing planning' (ibid:190) can be quite challenging. However, the progressive nature of PCT (Tomlinson, 1995) coupled with flexible scaffolding is not only beneficial for mentees but it also gives mentors a stimulus and time for reflection in and on their own classroom teaching. Co-teaching provides mentoring moments during the lesson as opposed to post-observation feedback meetings when the action has already taken place. Mentees need practical advice to help them improve their teaching practice and that is possible when mentors intervene during the lesson and modify instruction then and there making it easier and more accessible for mentees to apply those changes in practice in real time.

It is almost impossible to imagine mathematically resilient students if the teachers themselves are not resilient and gritty professionals. Teacher resilience affects students' engagement, motivation, and academic achievement. Johnston-Wilder & Lee define mathematical resilience as 'a positive approach to mathematics that allows people to overcome any affective barriers presented when learning mathematics' (2010:1). This is also true for learning how to teach. Mathematics teachers need to have that positive approach to learning how to teach better. Johnston-Wilder et al. (2013) argue that one of the four factors of mathematical resilience is being aware of and recognising the support available from peers and mentors. After the support has been arranged, it is important to ensure that the learning environment consists of essential attributes e.g. 'trust, courage, articulation, collaboration and persistence' (ibid). This kind of environment encourages mentees to 'feel motivated and appropriately supported' giving an opportunity 'to enter their growth zone' (ibid).

Mentoring through co-teaching (MTCT) model

The MTCT Model is flexible and can help support and empower mentees (Figure 1). The mentee plans a lesson and, in a reflection-before-action (Schon, 1987) meeting, both predict possible challenges and make any necessary changes to be better prepared for the lesson. The most important stage in this model is when co-teaching takes place. The mentee is the main teacher here and as a result of reflection-in-action (ibid), the mentor intervenes – very discreetly and only when there is need to intervene e.g., to clarify instructions, ask differentiated questions, add to the answers given by the mentee or alert the mentee to things they might have missed (Brookfield, 2017:66). In addition, the mentee has the reassurance that somebody is watching their back, pedagogically speaking, in case an activity goes wrong (ibid:137). Most importantly, this is when mentoring in the moment (Marciano et al., 2019) comes into action and the mentee has an opportunity to see how teaching challenges are immediately responded to by the mentor then and there.

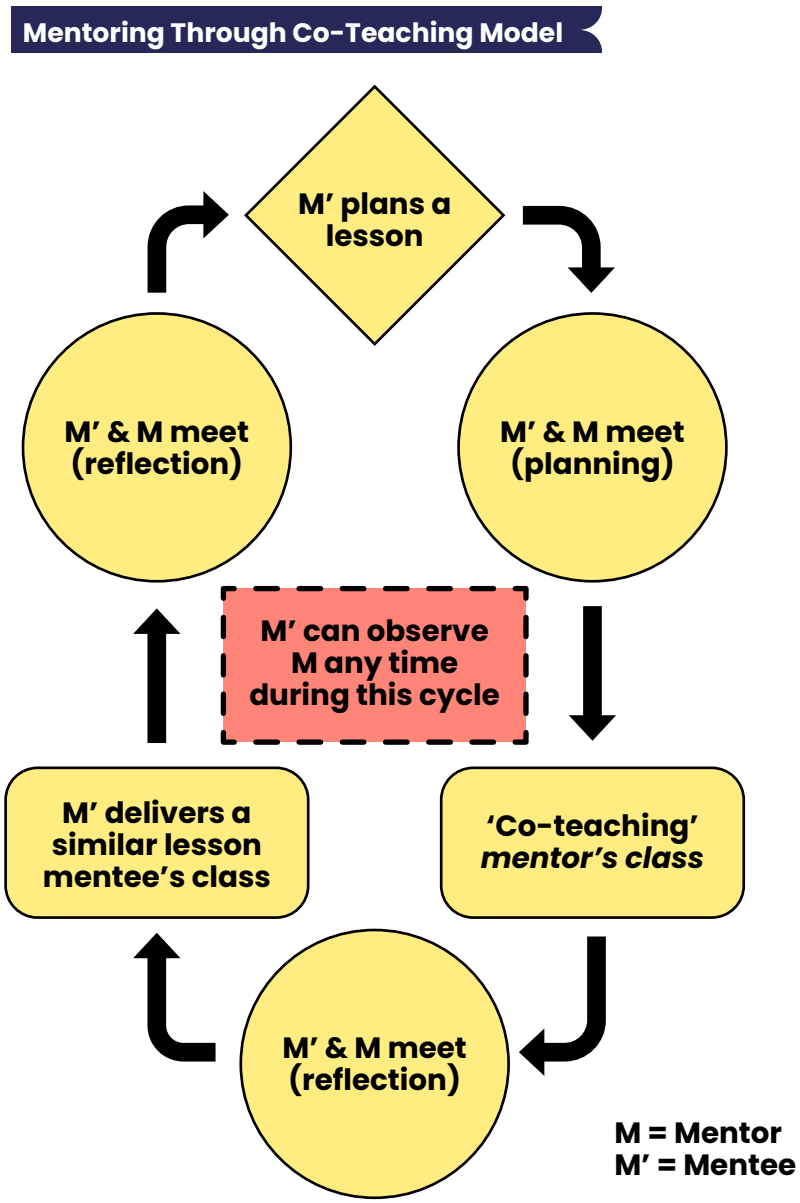

Figure 1: Mentoring Through Co-Teaching (MTCT) Model

After the co-taught lesson, the reflection-on-action (Schon, 1987) meeting takes place where both can offer different perspectives on a class, interpret classroom events in multiple ways, help each other recognise assumptions, and offer helpful analysis on what went well and how it can be even better (Brookfield, 2017:136). In the next stage, the mentee makes some minor changes to the lesson, if needed, before they teach a similar lesson to their own class using their own flavour and hence learn from their own experience. This is followed by a reflection-on-action (Schon, 1987) meeting in which both measure the distance travelled by discussing how the lesson went and what changes the mentee would like to make in the future.

The mentee is free to opt for an observation when they teach their class, but this is not mandatory. Since this model is flexible, some of the stages can be skipped if necessary and the mentee is not observed if they do not wish to be observed. It should also be noted that the mentee has the freedom to observe the mentor at any time during this cycle so that they 'acquire action strategies' while 'being there' which 'can be inestimably richer' (Tomlinson, 1995:47).

Research Methods

I present a case study for this research, because it can offer general relevance to professionals by providing vicarious experience which can lead to naturalistic generalization. It can also be more relatable to teachers' personal experience and potentially be relevant and beneficial. I used my own direct observations which might be dismissed as 'hopelessly subjective and impressionistic' (Brookfield, 1995:198). Also, observation reports from three different observers helped me investigate at a greater depth and see my assumptions with more clarity (ibid:200). The reports included the observers' reflections of 'their own interpretations and reactions' guiding me to 'new ways of thinking and acting' (Brookfield, 2017:68). Towards the end of the mentoring programme, a semi-structured interview was conducted with the participant. The interview questions were shared with the participant beforehand so that they could give in-depth answers. Moreover, the participant completed three self-evaluation reports during the mentoring programme providing their own reflections and insight. The learners' voice was also used to add in their experience and perspective of co-teaching in class.

During my research journey, studying theory and relating my own experience to those of others – the fellow travellers, whom I met either face to face or through reading their research – helped me identify an effective mode of travel to reach my destination and gave me a detailed interpretation of mentoring through co-teaching.

Data Analysis and Discussion

Thematic text analysis was used to analyse the data because it is 'a flexible and useful research tool' which can 'provide a rich and detailed, yet complex account of data' (Braun & Clarke, 2006:5). Themes are generated inductively from the patterns in the data which is presented in the form of cohesive snapshots that elaborate the data.

Theme 1 – Freedom

Having the freedom to be able to try out new and innovative ideas in a classroom helped Sara, my mentee (a pseudonym), feel more confident and creative. Sara said that she had the freedom to pop into my lesson whenever she felt the need and this helped her see in action how we can approach a variety of resources. She reflected on the stages of the lesson (Schon, 1987) and then taught in her own way using similar resources, making changes to 'only small aspects at a time' (Tomlinson, 1995:73). This sense of freedom helped Sara to be more creative and innovative. She made her own choices for her own teaching situation, adding to the developmental aspect. Sara said that in the co-taught and subsequent lessons, she felt free to try out her own ideas and I believe that this sense of freedom brought her confidence back.

Theme 2 – Trust

Establishing a good rapport through non-judgemental and completely developmental observations encouraged and motivated Sara to continue the mentoring programme. Having that higher level of trust in the programme made Sara feel comfortable throughout her learning journey. I did not evaluate the performance of my mentee. If that had happened, it would have spoiled our relationship.

Theme 3 – Learning together

Sara was fully engaged and immersed in being mentored, which helped me develop my own professional expertise by allowing me an opportunity to reflect in and on my own teaching (Tomlinson, 1995). However, just learning together, in my opinion, is not enough. Making the mentee aware that their teaching ideas are valuable, and the mentor is also learning from them is crucial. While I observed Sara a few times, I made a note of the teaching techniques she used. I successfully tried these ideas and techniques in my lessons. When we met for our feedback/ reflection meeting, I gave her vivid examples of how I had incorporated her strategies into my own teaching and how that benefited not only me but also my students. As a result, Sara also shared those techniques and ideas with other maths teachers in her department.

Theme 4 – Transformation

A transformation in one's behaviour and culture cannot take place quickly; it can take time to bring about transformations in teaching practices. As is evident from the analysis above, mentoring through co-teaching proved transformational not only for Sara but for me as well. Sara felt free to try out creative and innovative activities in her lessons. I, by being part of this mentoring programme, have discovered how co-teaching can make mentoring more effective and transformational. As a result of the mentoring and support, Sara feels she has become empowered and confident as well as 'autonomous and agentic' (Hobson, 2016:19).

Theme 5 – Self-efficacy and resilience

Sara's self evaluation forms (Figure 2) show that she has improved her questioning technique and she also feels better able to answer her students' questions. Towards the end of the mentoring programme, Sara's sense of feeling supported was also restored. She said that this has happened through watching the mentor in action during the co-taught lessons. Sara's formal observation reports show that the efficacy of her teaching has improved.

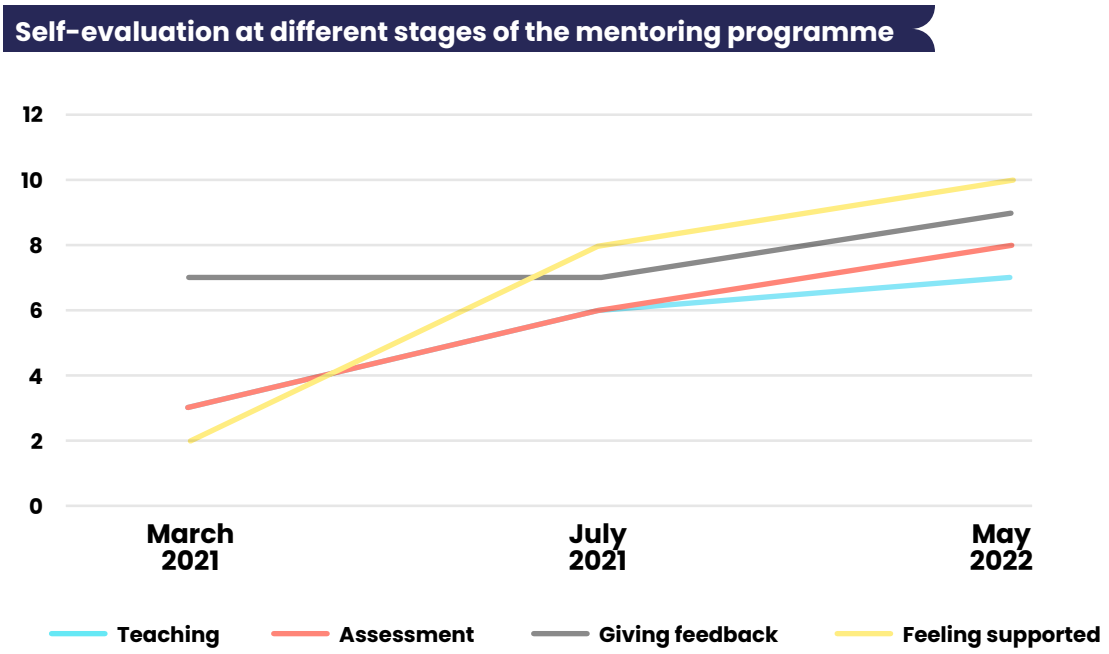

Figure 2: Self-evaluation reports completed by Sara

The guidance and mentoring that Sara has received through co-teaching have helped her become a more resilient teacher due to the strengthening of her sense of self-efficacy (Day & Gu, 2013). This transformation was possible because she received the support that she needed to become more resilient (Johnston-Wilder et al., 2020). I agree with Sara that *'being guided brings the best out of the people'*. The observation reports also corroborate this: 'creative and engaging resources'; 'students are relaxed and engaged throughout'; 'students are learning from each other'. In this programme, it was my role to help my mentee 'see the good things, the really valuable things' in their own teaching practice (Marciano et al., 2019:142).

Sara's students also provided their perspective of co-teaching using positive words e.g. 'beneficial; good; great; better; fantastic; supportive; extra help; helpful'. When asked to rate from 1 to 10 (10 - highest) the usefulness of having two teachers in class, two of them gave a score of 10 each and others rated the usefulness at 8.

Conclusion and recommendations

To conclude, developing teacher resilience can be done by providing the right support. I believe that mentoring through co-teaching is one of the most efficient ways whereby teachers can be supported in order to grow professionally and develop a sense of self-efficacy. Allowing mentees some flexibility gives them the time and space they need to become more resilient by making use of individualised support at their own pace. Reviewing is also important during the reflection meetings, as well as providing a 'safe learning environment' (Johnston-Wilder et al., 2013). Non-judgemental and developmental feedback can be shared informally, reviewing the mentee's progress and helping them feel more empowered and confident.

- I make the following recommendations so that mentoring through co-teaching is beneficial.
- Use PCT joined with MTCT as part of induction to help new teachers become more resilient and confident early in their careers.
- Use caution as well as careful planning while pairing mentors with mentees.
- Ask for formal consent from mentees prior to the mentoring programme. There has to be a desire and willingness to learn and improve (Marciano, 2019:145).
- Provide the time and space for mentoring (Hellier & Davidson, 2018).
- Use the Advanced Mentoring Course (ETF) as minimum training for all mentors.

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Paper 2

People are afraid of looking incompetent: How can we stop fear of mathematics from holding them back?

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Abstract

Mathematics anxiety is a prevalent problem, reported by about a third of students globally. Recent studies suggest that mathematics anxiety is negatively related to individuals' ability to make advantageous choices and decisions beyond classrooms. There is a need for simple, comprehensive, practical approaches to overcome emotional barriers to learning mathematics. This paper discusses the results of a qualitative small-scale study implementing 'The Toolkit', an approach introduced by Johnston-Wilder et al. (2020). The research underpinning The Toolkit is reviewed, the notion of psychological safety introduced, and the process of the intervention described. The results report reduced levels of anxiety and development of positivity towards mathematics. The key themes highlight the importance of addressing emotional aspects of learning mathematics in the classroom. All participants rated the Toolkit as 'extremely useful' and would advise its use as a practical strategy to help anyone overcome mathematics anxiety. This approach warrants further research to properly establish its efficacy in different contexts.

Key words: affective barriers, mathematics anxiety, mathematical resilience, MR Toolkit, psychological safety.

Introduction

Poor numeracy is widespread in the UK. Government statistics suggest that 17 million adults in England - half of the working-age population - have everyday mathematics skills roughly equivalent to those expected of a primary school child (Skills for Life Survey, 2012). Poor numeracy affects work productivity, people's income, health and wellbeing. Although many efforts are in place to improve numeracy in the UK, efforts often focus on the role of the cognitive factors (such as increasing mathematics requirements, introducing novel teaching strategies and increasing educational standards), while the emotional and social factors of improving numeracy and mathematics achievement are mainly overlooked (Ofsted, 2021).

Emotional factors are known to affect mathematics learning, performance and interest in pursuing careers that require numeracy skills (Beilock & Maloney, 2015). To address emotional factors, mathematics educators and learners need an approach that can be integrated into the existing cognitive framework of learning mathematics and remediate factors negatively influencing mathematical performance of mathematics-anxious individuals (Chang & Beilock, 2016). Further education (FE) colleges in the UK face the challenges of delivering mathematics as a compulsory subject for those that have previously failed to achieve grade 4 in GCSE mathematics. The most commonly reported issues are low levels of motivation and engagement, affective difficulties such as low confidence and anxiety (Noyes & Dalby, 2020) and poor attendance. Seligman (2007) considered that repeated exposure to stress can lead to avoidance in order

to escape repeated psychological suffering. Johnston-Wilder et al. (2013) propose that the accumulating effect of the repeat negative experiences in maths can result in avoidance as a form of psychological protection from further harm.

Johnston-Wilder and Lee (2010) defined 'mathematical resilience' as a framework to counteract mathematics anxiety and helplessness. Mathematics resilience (Fig. 1) involves four factors: growth mindset, personal value of mathematics, need for effort and struggle to achieve learning, and accessing available support. Johnston-Wilder et al. (2020a, p.1) introduced the Mathematical Resilience Toolkit in order to develop resilience and overcome anxiety. It combines four tools aimed at establishing psychological safety and an effective response to challenge and feeling of threat in the context of learning mathematics. In this study, the effectiveness and accessibility of the Toolkit in building resilience, reducing mathematics anxiety (MA) and increasing the effectiveness of learning are explored in the context of adult learners in a Further Education college in London.

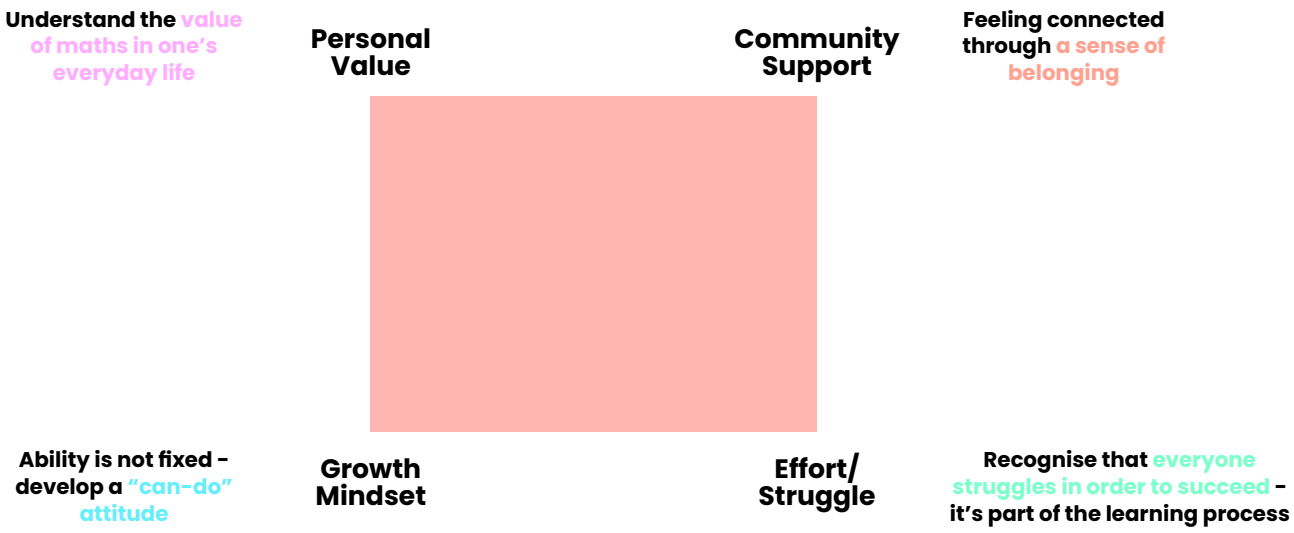


Figure 1 Mathematical Resilience

Methodology

Approach

This small-scale action research study was undertaken with eight mature students studying for a GCSE in mathematics. Two researchers (Sue and Masha) collaborated throughout the process, collaborating on the structure and timeline of the intervention and the data interpretation. One of the researchers (Masha) was their mathematics teacher. The approach was to study the effectiveness of the intervention from more than one perspective, providing a holistic and critical interpretation. Data was gathered in the academic year ending in June 2021, from students, teacher, academic researcher, and the wider community. Qualitative data were collected through field notes, semi-structured individual or small group interviews and students' mathematics stories.

Intervention components

Listening to prior experience: In the first session, after introductions and procedure agreements, each participant was invited to share their personal maths story, good and bad experiences, and their coping strategies. All participants identified negative past experiences learning mathematics; some described painful experiences, e.g., "In class all other students understood the topic, and I would still not understand. It made me very sad and discouraged" and physiological elements, e.g., 'heart beating very fast', 'feeling dizzy'. Two participants remembered being physically punished by teachers for not understanding and would miss their classes to avoid punishment and shame.

Psychological education: The following session involved helping learners understand mathematical anxiety and the role of the brain and emotions in learning. The learners were presented with examples of connections between negative feelings about mathematics and low confidence and explored the effect of safety on learning. All learners confirmed that MA was a new concept to them and that they were not aware that fear of mathematics can disrupt their learning by overloading their working memory and causing mathematics-avoiding behaviours (Dowker et al., 2016). Instead, many learners had established beliefs in their inherent inability to learn mathematics.

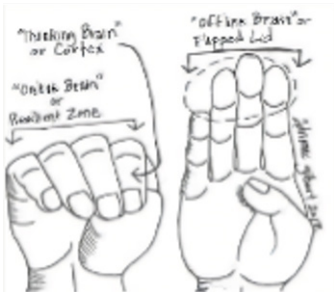
The Toolkit: The Toolkit comprises four tools (Fig 2): the hand model of the brain, the relaxation response, the growth zone model and the ladder model (based on Bruner, 1966). The tools were integrated into maths lessons and students were encouraged to practise managing anxiety and developing persistence outside the classroom.

Toolkit

The Growth Zone Model (GZM)
(Lee & Johnston-Wilder, 2018)

The Ladder Model
(Johnston-Wilder et al., 2020)

The Relaxation Response
(Benson, 2000)



The Hand Model of the Brain
(Siegel, 2020)

Figure 2 The Toolkit

Results

Thematic analysis (Braun & Clarke, 2006) resulted in four key themes, reflecting students' experience of the intervention: (1) the students, (2) emotions, (3) curriculum, and (4) teacher. Each theme was subdivided into subthemes summarised in Figure 5

All subthemes that indicated increased resilience were grouped together and coloured green, while those with decreased resilience were grouped together and coloured orange. We arranged the themes and their associated subthemes from internal to external to reflect the relationships and interactions between them.

Role of the students: When asked to evaluate the tools in the Toolkit most of the students spoke about the positive impact that Growth Zone and Ladder models had on their beliefs about the role of learners in the learning process, and specifically in developing self-efficacy. They commented that they had avoided challenge as a negative aspect of learning, believed that mathematical ability cannot be developed and felt helpless and hopeless in the face of challenge. The GZM provided the students with a model of the barriers to overcome and opportunities which they can use independently to understand their current learning and plan how to progress. They also reported feeling more in control and reminded themselves that growth is possible. One said:

> *"Now, I use Growth model at work, it helps me to come out of the comfort zone and strive for more with confidence. I even share it with my colleagues, when they go for promotions" (Mathilde)*

The Ladder Model reminded them of the need to seek specific support; the students evaluated it as a useful tool encouraging them to break down difficult questions into manageable tasks. They concluded that having the tools gave them a set of "go to" strategies in the face of difficulties of maths, which had a positive impact on their confidence to have an active say in their maths learning.

Decrease in resilience Subthemes	Themes	Increase in resilience Subthemes
Negative view of challenge	Theme 1: Students	Value of challenge Deliberate effort Growth mindset
Fear and danger Negative about future	Theme 2: Emotions	Feeling safe Positive about future
Fast pace High curricular content Lack of support	Theme 3: Curriculum	Slower pace Less content at a time Varied support
Criticism Strict Poor teaching	Theme 4: Teacher	Kind Open to listening Good teaching

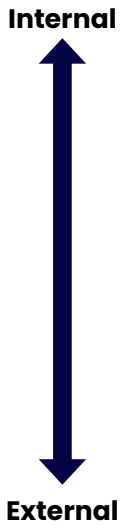


Figure 5 Themes and subthemes

Role of emotions

Students expressed that repetitive failure and apparent lack of ability to achieve a required pass grade for vocational qualifications or degrees increased their feelings of fear and hopelessness. In response, they opted for low paid and low skilled jobs. Challenge corresponded with fear, defeat and low self-esteem followed by avoidant behaviour. When asked about their psychological safety and discussions about the impact of emotions on learning, they reported that speaking about emotions in maths lesson was a new experience:

> *"Talking about how I feel in maths class seemed very unusual way to learn maths. I thought to give it a go and to my surprise it helped me learn!"*

The learners valued the Toolkit highly as a practical strategy for addressing fear and hopelessness.

> *"I used to see mixed up numbers in maths problems and felt really stupid. I am not stupid, my brain can' t learn when I am scared" (Anthony).*

The GZM allowed students to distinguish between the range of emotions from nervousness to fear and to communicate those emotions without feeling stupid.

> *"When I am stuck doing MathsWatch at home, I can ask myself - what zone I am in, and when I am in the RED, I know that I am stressed, I need to relax, do something else and then come back to my task. I often can do my tasks very well after that as I start thinking clearly" (Seye)*

When asked about their view on the relaxation response, the students commented that was a refreshing break 'like a reset' when they felt overwhelmed by mathematical rules and numbers. They commented that fear of failure and shame negatively impacted their mathematical abilities and positive affirmations encouraged feelings of hope in their progress in mathematics.

Role of the curriculum

When asked about the elements of learning that had negative impact on their confidence and safety, feelings many students associated those with fast pace of going through curriculum, high saturation of content and hopelessness when then fell behind due to fast pace. One participant said:

> *"it feels like you are running fast but teacher is moving even faster. So that your fast is never fast enough in maths"*

The participants expressed the feelings of 'left behind' when everyone else moved on; they used to believe there is something wrong with them as everyone one else seemed getting on with the pace which prevented them from reaching out for help.

> *"I always was scared of being behind and not catching up with other students. It made it only worse. This year was different because I knew I can always reach out for help"*

Role of the teacher

The students noted that teacher's role in facilitating the intervention during maths lessons was very important. The students found it valuable that the teacher came across with empathy rather than critical and judgemental. They added that teacher was sensitive to their feelings, fears and worries, and they felt that the teacher considered their emotional difficulties with maths in lesson planning. They felt validated and safe to continue to open up and speak about their experience. They expressed that support was explicit and they felt comfortable asking for help.

> *"I always was scared of being behind and not catching up with other students. It made it only worse. This year was different because I knew I can always reach out for help" (Ariana)*

Discussion and Conclusion

This study addressed barriers to learning mathematics by explicitly embedding a set of tools (Johnston-Wilder et al., 2020) for mathematical resilience in a mathematics classroom. A unique collaboration between one of the authors of mathematical resilience, Sue Johnston-Wilder, and a mathematics teacher, brought the experiment into a FE classroom by implementing psychological safety as core and constructing mathematics teaching around the value of resilience, growth and safety. The teacher had high intrinsic motivation to extend teaching mathematics beyond standard cognitive methods of delivery and engage in practitioner research.

The intervention was conducted with a group of learners and results were analysed over time to examine the effect in the context of regular classroom teaching. The data conveyed a positive shift in students' attitudes towards challenge and struggle. Students communicated that the tools were easy to use outside of class and reported their application in work-related contexts. They quoted that positive affirmations, relaxation and distraction strategies helped them manage their anxiety and fear of failure during independent work and assessments. All students in class attempted and completed homework and actively sought support from the classroom community and online resources; they shared feelings of empowerment and confidence in overcoming their own barriers and failures.

One of the themes highlighted the role of teacher's empathy and teaching skills. This intervention calls for further implementation of the Toolkit with other practitioners in similar settings to explore the effect of the teacher in developing mathematical resilience and the effectiveness of the tools with different practitioners.

This research was conducted to determine if the intervention was effective on a smaller scale before replication and expansion to other courses. The replication of the study with larger sample size is recommended. Student retention and achievement on mathematics courses is a crucial goal in further education institutions and it significantly predicts sustainability and success of further education sector. Notably, mathematics anxiety and avoidance are powerful psychological barriers affecting many students' vocational performance in FE (Tobias, 1991), which warrants further research on effective interventions that are accessible to students and educators (Lyons & Beilock, 2013).

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Paper 3

Developing mathematical resilience while addressing mathematics anxiety

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Abstract

Recent international studies are alarming about mathematics anxiety (MA) levels and mathematics performance in French schools. As a mathematics teacher in disadvantaged areas in France, I observed avoidance and passivity, symptoms of MA. As a researcher, I focused on developing mathematical resilience and coping skills. I surveyed students (N=1902) from disadvantaged areas in France aged 10-15 years about MA and teachers (N=185) about students' attitudes toward mathematics. Using a design-based approach, I designed a whole-class intervention, based on 3 tools: hand model of the brain, relaxation response and growth zone model. I measured the impact on MA, using pre- and post-intervention surveys (N=120). A quarter of participants declared high MA. Visible symptoms, avoidance and passivity, were observed by most teachers surveyed. This study, based on the affective domain, adds ways of developing mathematical resilience while addressing MA. The intervention proved effective, especially for students reporting high MA.

Keywords: mathematical resilience; mathematics anxiety; mathematical coping skills; intervention.

Introduction

As a teacher of mathematics in disadvantaged areas in France, I observed symptoms of mathematics anxiety and self-inefficacy. Moreover, worldwide education studies show that French learners are significantly affected by mathematics anxiety (PISA, 2012), reduced mathematics performance (TIMSS, 2020), indiscipline in classrooms (PISA, 2019) and social inequality (PISA, 2015). Therefore I focused on developing mathematical resilience and constructive coping skills.

Key concepts

The key concepts of this study are concepts from psychology that have been adapted to the context of teaching and learning mathematics.

Mathematics anxiety

Mathematical anxiety has been defined as a feeling of tension and fear triggered by manipulating numbers and solving mathematical problems in everyday life and schooling (Richardson and Suinn, 1972) and MA and mathematics performance are known to influence each other (Carey *et al.*, 2016). Mathematics anxiety can be described as low, medium and high mathematics anxiety (Mahmood and Khatoon, 2011). MA is constituted by specific affect, cognition, such as self-inefficacy thoughts, and actions, such are avoidance and paralysis (Bandura, 1988).

Mathematical resilience

Mathematical resilience is a positive attitude vis-à-vis mathematics, allowing students to overcome the emotional barriers learning mathematics (Johnston-Wilder and Lee, 2010) can induce. The construct of mathematical resilience has four aspects (Lee and Johnston-Wilder, 2017). First, maintaining a growth mindset, so that learners believe that their mathematical skills can be developed through persistence and work. Second, knowing that mathematics can have personal value, have a value in the world and that the learner is valued in their learning. Third, accepting some difficulties inherent in learning mathematics. Fourth, knowing how to find the appropriate support to continue to learn despite encountering difficulties.

Mathematical coping skills

Mathematical coping strategies are the cognitive processes used to respond effectively to a situation perceived as challenging or threatening (Lazarus and Folkman, 1984) to well-being in a mathematical context. Coping strategies can be constructive or of limited value.

Research questions

I set out to address three questions, mainly in the context of secondary schools in disadvantaged areas schools in the suburb of Paris:

Q1: Which learners report mathematics anxiety?

Q2: Are symptoms of mathematics anxiety observed by the teachers?

Q3: How can a whole-class intervention address mathematics anxiety and develop mathematical resilience for student?

Methodology: mixed method

A paper-based students' survey was used to measure MA (MAS; Betz, 1978) with participants aged 10-15 (N=1902) in disadvantaged areas in Paris. An online teachers' survey involving mathematics teachers in France (N=185) was used to study teachers' perceptions of students' attitudes toward mathematics. 25% of the participant teachers work in disadvantaged areas. Also a design-based approach was used to plan a whole-class intervention, based on the three tools (Johnston-Wilder et al., 2018) previously mentioned. Pre- and post-intervention surveys (MAS; Betz, 1978) were used to measure any impact on MA (N=120).

Results

Learners' survey

On average, 25% of participants reported high MA. The proportion of learners reporting high MA increases significantly with age. However, there is no significant gender difference.

Teachers' survey

A majority of teacher participants observed avoidance, passivity and learners easily discouraged, all of which are symptoms of MA and lack of mathematical resilience. Avoidance was observed by a majority of teachers (58% of participants; 68% of participants in disadvantaged areas). Passivity was observed by most of the teachers (92% of participants; 91% of participants in disadvantaged areas). Learners easily discouraged was also observed by almost all teachers (95% of participants; 93% of participants in disadvantaged areas).

Intervention

The aim of the intervention is to develop mathematical resilience while addressing mathematics anxiety by providing mathematics-related constructive coping skills, both problem focused and emotion focused. This intervention is based in the work of Dweck concerning Growth Mindset (Dweck, 2017). The intervention constitutes complementary tools: the mathematics resilience grid, the mathematics resilience card, posters and a short, silent relaxation video.

The Maths Resilience Card (MaReC)

The MaReC is a foldable square card to be filled in and manipulated by the learner. The card uses 3 tools adapted to the mathematical context by Johnston-Wilder et al. (2018): Growth zone model, hand model of the brain and a relaxation response.

The first tool is based on the Growth zone model, presenting 4 learning zones: boring (blue), comfort (green), challenge (yellow), threat (red). Each zone is defined by specific thoughts, behaviours and feelings.

The second tool is the hand model of the brain, a metaphor using the hand to represent the relevant parts of the brain. This model can represent the impact of high-level negative emotions on thinking. The learner may feel stupid when overwhelmed by negative emotions. In this case, the tool invites the learner to think "I feel stupid when I am panicking" instead of "I am panicking because I feel stupid".

The third tool is the relaxation response (Benson, 2000). In addition, other coping skills have been added to continue to learn mathematics from any zone. These coping skills invite the learner to accept the feelings and change the behaviours and the thoughts in a specific manner for each zone. In the threat zone, the learner is invited to calm down, in the challenge zone, the learner is invited to ask for help when needed, in the comfort zone, the learner is invited to challenge the work and in the boring zone, to make the work a little bit more challenging.

The card is a square divided in 4 parts, one for each zone. When folded, the card presents the four zones and the thoughts, feelings and behaviours associated. When unfolded for one zone, the card provides a list of constructive coping skills for the chosen zone.

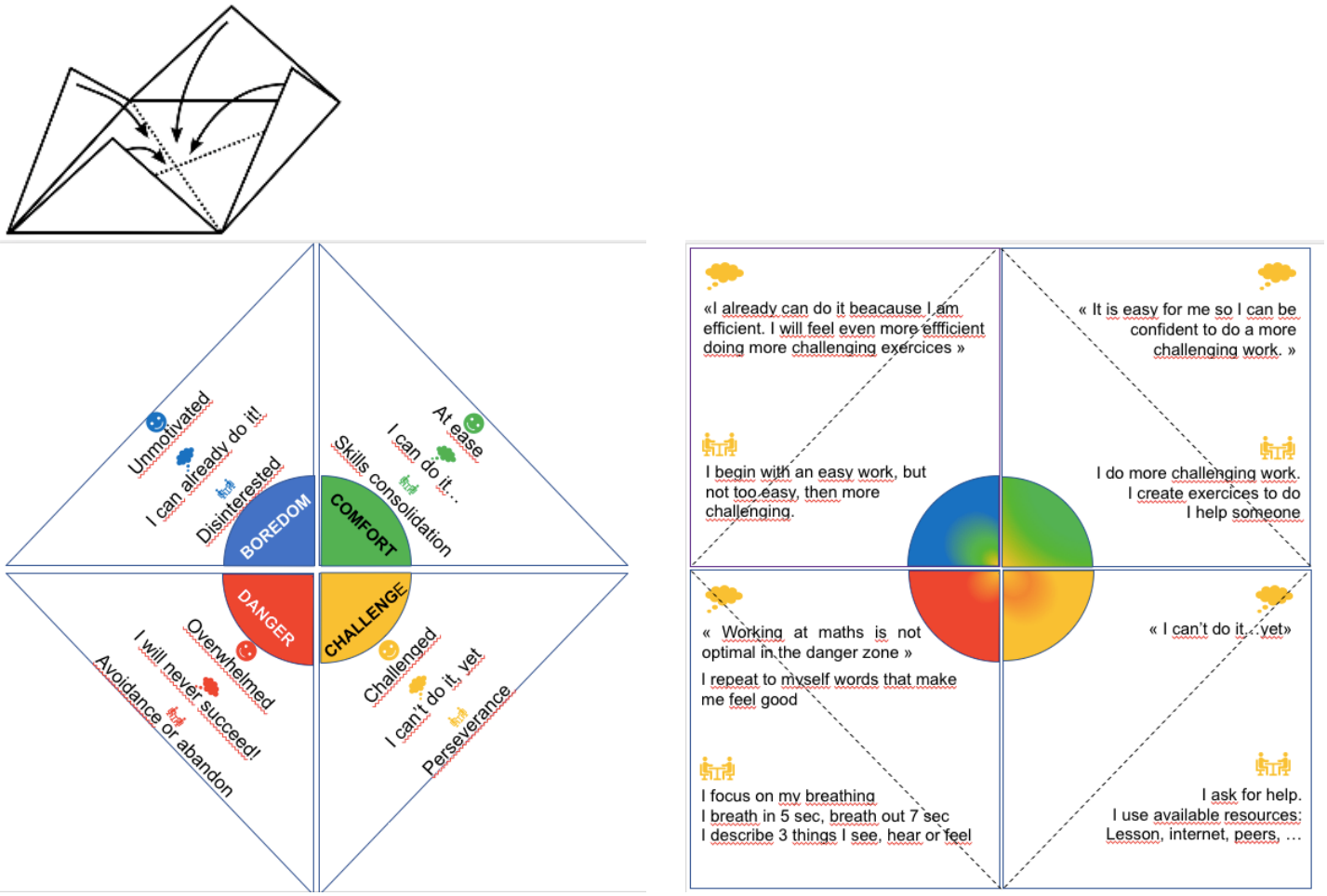
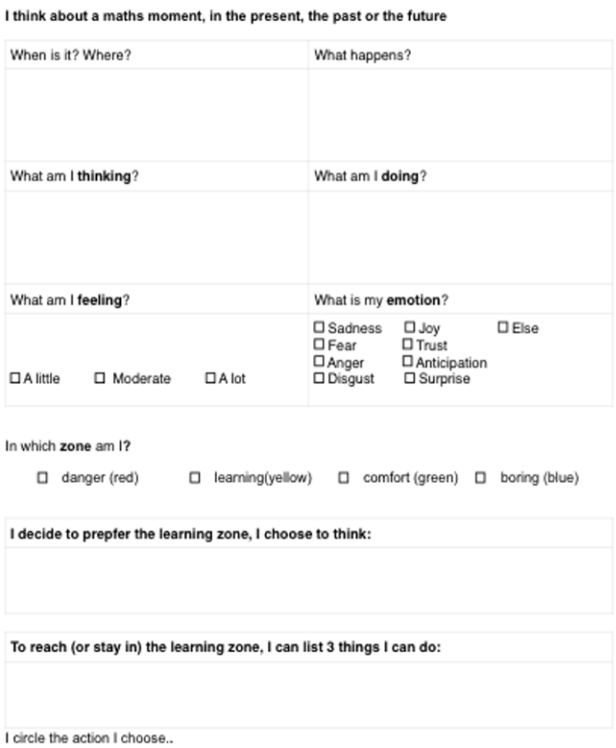


Figure 2: the Maths Resilience Card (MaReC) folded (left) and unfolded (right)

The Maths Resilience Grid (MaReG)

The MaReG (Figure 3) is a page to be filled by the learner. It can be a collective or individual task, but each learner reflects on a personal situation. The MaReG is used in association with the MaReC, alone, or with the presence of a teachers, carer or coach. Several MaReGs can be printed to constitute a booklet for the learner. The MaReG is constructed in 3 parts, inviting the learner to reflect on a mathematics situation, past, present and future.

The use of writing in the MaReG is inspired by the writing intervention described by Stogsdill (2013). Thus, if the learner chooses to reflect on a past and unpleasant situation, writing enables the learner to approach the past while containing their potential emotions. In addition, writing about potential mathematics trauma is a therapy exercise in itself. Additionally, the GROW model (Whitmore, 2017), a coaching tool, underpins the MaReG in that the users are invited to describe the Reality, remember their Goal, relate possible Options and be Willing to choose one action amongst the options.



I think about a maths moment, in the present, the past or the future

When is it? Where?	What happens?
What am I **thinking**?	What am I **doing**?
What am I **feeling**?	What is my **emotion**?
☐ A little ☐ Moderate ☐ A lot	☐ Sadness ☐ Joy ☐ Else ☐ Fear ☐ Trust ☐ Anger ☐ Anticipation ☐ Disgust ☐ Surprise

In which **zone** am I?

☐ danger (red) ☐ learning(yellow) ☐ comfort (green) ☐ boring (blue)

I decide to prepfer the learning zone, I choose to think:

To reach (or stay in) the learning zone, I can list 3 things I can do:

I circle the action I choose..

Figure 3: The Maths Resilience Grid (MaReG)

The first part enables the learner to describe a situation (Reality) by identifying facts, thoughts, emotions and behaviours. The list of the emotions proposed is the 8 basic emotions described by Plutchik (2001). The analysis of the mathematics situation is inspired by elements of cognitive analysis developed by Finkel (2022) enabling the identification of facts, emotions and actions in a scene strongly charged with emotion, and to restore it in the memory without the overly negative and destructive affects. The second part invites the learner to refer to the folded resilience card to find the zone associated with the mathematics situation described. The third part reminds the learner of the goal of the MaReG (continue to learn mathematics) and invites the learner to unfold the resilience card at the relevant zone and write their thoughts that may enable the continuation of learning. Then, the learner is invited to write the Options of behaviours available and choose to commit to one.

The Maths Resilience Posters

The posters are a synthesis of the MaReC designed to be displayed in the classroom. The use of posters is inspired by the stuck poster developed by Chisholm (2017). The aim of the stuck poster is to have visual in the classroom to refer to when the student feels stuck in mathematics.

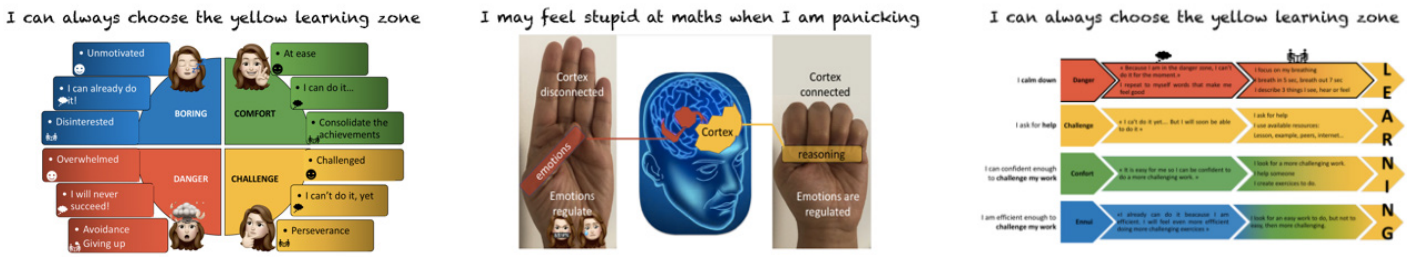


Figure 4: The Maths Resilience Posters, tool 1 to 3 (from left to right),

The Relaxation Video

The video is a short, silent video of the relaxation response (Benson, 2000) to relax from the threat zone. This video can be used in the classroom or elsewhere. As it is a silent video, this video can be projected in loop, in a whole class context, during a test for example, to allow a student to watch it if experiencing the threat zone.

Figure 5: screenshots of the relaxation video

Impact of the intervention

The design has been improved through observations, teacher interviews, learners' work, over 4 cycles. Cycles 1 to 3 of the intervention have been conducted in secondary schools in disadvantaged areas in France; pre- and post-intervention MAS questionnaire were conducted. The results are more promising for learners reporting high MA and younger secondary learners. On average, the MA score decreased 6.5 points for learners reporting initially high MA; the MA score decreased 2.8 points for first- and second-year secondary learners.

Conclusion

From a large-scale survey, with learners 10-15 years old, mathematics anxiety is prevalent in disadvantaged areas in France. No gender difference was observed but the situation deteriorates with age. Most mathematics teachers observe symptoms of mathematics anxiety in France. An intervention was designed, based on three tools, aiming to address mathematics anxiety and develop mathematical resilience in a whole class context. The impact is promising: on average, the intervention of previous cycles was effective in reducing the mathematics anxiety score of students with high mathematics anxiety and younger students.

Constructive coping skills, problem- and emotion-focused, should be added to the mathematics curriculum from the first years of secondary school. This study can help mathematics education in France become more effective and inclusive. Additional studies are needed about the impact of the intervention in France and in the United Kingdom.

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Further reading

Website about mathematical resilience: http://www.mathematicalresilience.org

3 tools Video in French: https://www.youtube.com/watch?v=UdpIG7y7OPg&t=33s

The relaxation video: https://www.youtube.com/watch?v=J5TeKfBhLHM

Paper 4

Step into a new dimension with Augmented Reality

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Abstract

The global pandemic has changed the mode of delivering lessons. Adult learners have missed and continue to miss the hands-on learning activity, which is a form of experiential learning (Kolb, 1984). When it comes to teaching three dimensional, geometric shapes (3D), learners often benefit from manipulating concrete shapes (Bruner, 1966). This study examined the use of an Augmented Reality (AR) mobile application, as a learning tool for teaching geometric shapes. Data was collected from thirty Learners. Learners were divided into Control and Experimental groups. Qualitative data was gathered using mixed methods of one-to-one interviews and questionnaires to evaluate the learners' experience. The data was triangulated with observation by the teacher. In this study, content analysis has been used to analyse the data. Results showed an improved activity and interactivity within the session. Further studies are required to evaluate quantitative improvements in student performance.

Key words: Augmented reality, mathematics, 3D shapes, experiential learning.

Introduction

''It's a fine idea, '' said the boy; ''who discovered it?''

''It is a fact that has always existed but is now utilized for the first time.''
(Baum, F., 1901).

Author of the Wizard of Oz, Frank Baum, prefigured the possibilities of Augmented Reality in 1901 but only the pandemic urged me to use it in teaching mathematics online for the first time. The move to remote teaching during the Covid-19 pandemic posed a challenge in teaching some geometrical mathematical concepts, as learners were missing the tactile element. I felt that moving away from abstract and theoretical teaching methods to a more interactive way through the use of new technologies may help address this issue. Keeping learners in the growth zone is supported by creativity and conjuring a sense of curiosity and exploration. Taking advantage of the ability of Augmented Reality (AR) to visualise 3-dimensional (3D) shapes and provide learners with an equivalent to the hands-on element, I will investigate the potential of using AR as a learning tool to teach 3D geometrical shapes online.

When it comes to learning the properties of 3D shapes, learners often benefit from handling the objects (Bruner, 1966) and manipulating them to identify their different properties. Since the first step of the concrete, pictorial, abstract learning sequence, proposed by Jerome Bruner, is missing in virtual classes, an AR mobile application was used, aiming to replicate this stage. As part of this research, thirty learners from Level 1&2 Functional Skills (FS) and GCSE Maths were divided into two groups: a control group and an experimental group. The control group used traditional material such as PowerPoint presentations and demonstration videos, whilst the experimental group used a dedicated AR app to learn about geometrical solids.

This teaching innovation will enable me to investigate:

1. How AR can be used to replicate 3D learning in the classroom
2. To what extent AR can improve learners' motivation and engagement

Ethical considerations

Ethical considerations were applied by ensuring the employment of approved procedures following the BERA's ethical guidelines (BERA, 2018). It was imperative to first obtain permission from my line manager to explain the nature of the intervention, when and where it is taking place. I also assured him hat I would adhere to all college policies on gathering data, give due regard to the safeguarding apparatus in place and to comply with the General Data Protection Regulation (GDPR, 2018). The intervention would also solicit informed consent from participating learners to use data collected alongside personal information to compare and evaluate impact. It was made clear that data will be stored securely and will be deleted after 12 months of use. In addition, I have clarified that participation is voluntary and that the learners are free to withdraw at any time, without giving any reason.

Literature review

It has been argued that learners often fail to make connections between real-life 3D objects and 2-dimensional (2D) shapes resulting in difficulties in distinguishing geometric solids from flat shapes (Gonzalez, 2015). This study aims to evaluate the ability of AR to enhance the spatial abilities and engagement levels of learners.

AR is a technology used to overlay reality with virtual objects in real-time and with the possibility of interaction (Azuma, R.T., 1997). Similarly, Billinghurst et al. (2012) agreed on the ability of AR to provide an interactive element by adding virtual information to the user's physical environment and allowing the user to interact actively with the content. Drawing heavily on the work of Dewey et al., Kolb (1984) helped develop the modern theory of experiential learning. Experiential learning involves observation and interaction with the learning material. It entails a hands-on approach to learning, where learners are actively involved in the learning process and make discoveries first-hand, instead of looking at a PowerPoint presentation or watching a video.

The Concrete, Pictorial, Abstract (CPA) approach proposed by Jerome Bruner in 1966 as a means of scaffolding learning consists of three stages. The fundamental concrete stage involves providing learners with physical manipulatives to explore the structure, find commonalities and rehearse the mathematics; the pictorial stage involves representing the concrete situation using drawings or images by making the cognitive link between the concrete resource and the picture; the abstract stage involves the use of words and symbols to communicate mathematically. Hence, replicating this tactile experience using AR in virtual mathematics classes seems to replicate a fundamental step when teaching abstract geometrical concepts online.

According to Torok et al, (2017), AR will become a very important educational tool in the future, allowing learners to deepen their understanding of core mathematical concepts via active interaction and illustrative demonstrations. They give a promising vision of AR technology as the future not just for education but for many other industrial fields.

The learner is the focus of this experience. The AR innovation is designed to provide a way for the learner to be able to explore, manipulate and handle the shapes; in order to provide an improvement in understanding through the interaction with the objects. Furthermore, Shin, Dunston and Wang (2005) linked movement and activity to spatial imagination, stating that participants who moved around the 3D model benefited significantly more than participants who only moved the model.

In light of the literature, it can be stated that AR has significant potential in mathematics education (Kellems et al., 2020).

Research Methods

The mobile application and its intended use in the classroom were introduced to learners, managers and tutors in the same institution. As the whole research was conducted virtually, a combination of qualitative research methods was used.

Learners were divided into control (n=15) and focus (n=15) groups to compare outcomes.

Stage one - the experimental step

The control group used traditional teaching materials such as PowerPoint presentations and tutorial videos to study the topic by seeing a 2D representation of the 3D objects, whereas the focus group used the 3D shapes app, which enabled them to manipulate, explore and walk around the virtual image of the shapes.

Stage 2 - gathering feedback

A questionnaire was used after the experiment to evaluate learners' satisfaction level with the experience, as well as engagement level and understanding of 3D shapes. Using Likert scales, the questionnaire included two aspects of Cognitive Usefulness (to see how helpful the app was in learning the topic) and Cognitive Easy-to-use to investigate how easily learners could use the learning tool. For example, on a scale of 1 to 5 (1-poor and 5-outstanding) rate the ease of using the different functions in the app; rate your experience of using the 3D shapes geometry app; rate how this experience had supported your understanding of the 3D shapes topic.

Stage 3 – interviews

Virtual interviews were conducted with learners, managers, subject leaders, and tutors in the science and maths department to evaluate the effectiveness and potential future use of the AR tool. I triangulated this with observation and note-taking to evaluate the engagement and interaction level of the learners.

Data Analysis

Since the use of AR in teaching mathematics online is relatively new, I will follow an inductive approach in analysing the qualitative data gathered to evaluate the benefits of using an AR smartphone application to teach geometrical shapes virtually. I will follow a thematic analysis approach to look for patterns in learners' feedback accompanied by content analysis to provide quantitative thinking by grouping the data and comparing frequencies.

Ease of use and operation of the app:

The utilisation of 3D shapes app was perceived as easy to use - 13 of 15 learners had no issues downloading the application and rated it as 'very easy', two learners rated it as 1 out of 5 on the Likert scale and the reason was identified as lack of IT skills. Learners consistently seem to find the application features to be well designed although one learner from the GCSE group had trouble operating the app.

Learners were asked if they had used any similar application previously. 9 out of 15 had never used such apps to learn 3D shapes but were happy to discover the new technology. Six learners had previous experience of using AR apps, however it was identified during interviews that these were mainly in furniture shops and video games.

Understanding and engagement:

When questioned about their experience of using AR within the session to learn 3D shapes, all learners at L1 and 2 rated their experience as very satisfactory or outstanding. Two learners at the GCSE level rated the experience as satisfactory, 2 as unsatisfactory, and 1 as outstanding. Some learners at L1 and 2 described the experience as 'fascinating' and 'interesting'.

Most GCSE learners reported that the experience was helpful to visualise the shapes and consolidate their knowledge of shapes, vertices, and edges. However, these learners found that the features of the AR application used were "very basic" and "repetitive". One learner rated their understanding level as outstanding, two learners as satisfactory, and two as unsatisfactory.

Looking at the engagement level, all learners at L1 and 2 rated the experience between 4 and 5 on the Likert scale; three out five GCSE learners rated it as poor and unsatisfactory, they described it as 'primary level content', 'basic' and 'time-consuming'.

Some learners were self-motivated to use the app outside the classroom and extend its benefits to family members such as children. One learner commented: '*It is very easy to navigate, my 10 year old son loves the app too, you can move the shape anyhow and it looks so real as though you are next to the shape.*'

Follow-up interviews analysis:

Follow-up interviews were conducted with participant learners. Learner A was 'fascinated' with the experience of some changes to the app such as adding more colour options and making the names of the shapes easier. Some learners used the app with their children and siblings. Other learners found the experience helpful in bringing the shapes to life and being able to manipulate them which improved their spatial abilities. All learners agreed that some features of the app need changing such as the Americanised names of the shapes and the excessive number of shapes available.

Interviews revealed that mathematics tutors were interested in using AR in online teaching. However, they reported that the main barrier was the cost of the app. One manager offered to purchase the license for tutors to use in the classroom. Another was keen to develop a similar app themselves. All tutors agreed that an app should be free and should work on iOS and Android devices.

Email consultations with the head of service, the manager of quality and improvement, and other senior staff members, were sent to introduce the concept of AR in education and its potential future use. All showed interest in the research and the majority had little previous knowledge of the area.

Teacher observation

Data analysis was triangulated with teacher observation to evaluate engagement levels. The results of the tutor observation revealed that learners in the experimental group were active and focused on what they were doing on their mobile screens. Instant interaction with the shapes was observed, as well as an improved participation level, where learners were answering questions immediately and accurately. One learner managed to print the net of a square-based pyramid and build it. On the other hand, most learners in the control group had their cameras off, so engagement and interactivity levels appeared to be significantly lower than those who used the app.

Findings

The implementation of the AR app in teaching geometrical topics was simple and participants used their smartphone devices to purchase and download the app. Only learners with iOS smartphones were able to take part in the experiment. The app was accessible anywhere and anytime, which enabled learners to use the app outside the session too.

The results indicated that the use of AR in an online session created an element of fun and excitement for most learners. However most GCSE learners were not interested and found the experience time-consuming. An increased element of activity and interactivity was observed by the tutor in the experimental group compared to the control group. Furthermore, the results revealed that AR supported learners' understanding through the manipulation of the 3D object and the exploration of the different properties and nets. It can be asserted that AR technology provided learners with alternative concrete objects to explore, which has increased their visual thinking skills.

It was also noted that some learners were self-motivated to use the app with their children or siblings. It can be asserted that AR technology made learning fun and increased learners' participation in the learning process.

In summary, it can be concluded that AR is a technology that provides learners with access to learning materials on demand and increases engagement levels. On the other hand, AR in mathematics has some disadvantages such as the resistance of some learners to using it, the cost of the applications, and the lack of IT skills for effective use. Overall, the use of AR is a promising trend in the teaching of mathematics.

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Paper 5

Developing Mathematical Resilience through dealing with errors: analysis of an intervention in Rio de Janeiro

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Abstract

Errors can be seen as reflections of not knowing but can also be understood as indicating knowledge that needs revisiting, analysis and reflection. In the context of the Vector Calculus course taught at CEFET-RJ (Federal Centre for Technological Education of Rio de Janeiro), it is very common for students to understand the concepts of the syllabus but have low performance because they do not recall the techniques required. In the period of remote teaching due to the pandemic, there was a need to use ICT in education. CEFET-RJ adopted Microsoft's Teams platform for classes to take place. An intervention case study to help to develop Mathematical Resilience (MR) in students was carried out in vector calculus classes in which students had to, in a collaborative way, create questions and solve them. This experience has been enriching, as it is an active methodology in which students analyze their own mistakes and collaborate with each other, developing mathematical resilience.

Keywords: ICT, Undergraduate Teaching, Mathematical learning difficulties, Errors, Mathematical resilience.

Introduction

There is no discipline in which the fear of error is so strong and rooted as in mathematics. It is evident that the demonization of error in mathematics has significant effects on an emotional level. Baccaglini et al. state that "... The spread of the fear of making mistakes, turns out to be the emotion most associated with mathematics ... the effects of the fear of making mistakes [result] in the block of thought processes". (Baccaglini; Di Martino; Natalini; Rosolini, 2018, p. 67).

In Brazil, many young people are leaving school very early. A recent education survey, the Continuous National Household Sample Survey (Pnad, 2019), released by the Brazilian Institute of Geography and Statistics (IBGE), reveals data on the worrying reality of schooling in Brazil. According to statistics, 40% of the population aged 25 and over have not completed elementary school, which represents about 53.4 million people. The rate helps to compose a broader scenario in which more than half of this population (52.6%) did not complete basic education, that is, did not graduate from high school, which corresponds to 70.3 million people. The Pnad 2021 already exists, but the data for this comparison are still not sufficiently clear as far as we know. What can be seen is that from 2019 to 2021 there was a slight improvement in several respects.

It may well be that in this context, mathematics is one of the subjects that causes young people to avoid school. The difficulties that students face in relation to mathematics extend over many years of their academic life and have several roots: the distancing of classes from the students' daily lives; the poor training of teachers, in addition to learning problems such as dyscalculia and attention deficit, among others. In addition to these issues there is the specific problem of the mistakes made by students in the classroom or in tests being regarded as an indicator of what the student knows or does not know, and often their relative intelligence, rather

than showing what they need to learn or reprise. Thus, analysing the answers given by students brings to both students and teachers the possibility of understanding how the students are appropriating knowledge and what needs to be done next.. This analysis requires depth, reflection, teaching experience and is a research methodology.

The analysis of errors as a teaching and learning methodology is reinforced by several authors (Cury, 2007; Krutetskii, 1976; Newell & Simon, 1972; Brousseau, 1983; Borasi, 1996) among others. Cury (2007) presents a new view of errors in her book "Análise de erros: o que podemos aprender com a resposta dos alunos" ("Error analysis: what we can learn from student responses"). According to the author, analysing the students' answers, besides being a research methodology, can be seen as a teaching methodology, leading students to question and build their own knowledge.

Moreover, the analysis of "errors" has been used to explore several issues in Mathematics Education, such as the analysis of teaching materials and teacher training. Cury (2007) argues that the students' output should be part of the pedagogical plan of institutions and the teacher's lesson plan. The author goes further:

> *the analysis of the students' productions is not an isolated fact in the teacher's practice; it is - or should be - one of the components of the institutions' pedagogical plans and of the teachers' lesson plans, considering the teaching objectives of each subject. But there are obstacles to its realization, which involve delicate aspects of teaching practice, since, being an assessment, it assumes the status of this, touching on feelings - feeling approved or rejected by someone - in memories - to have been criticized by someone to whom the student attributes authority or to have their ideas systematically accepted by the authority -, in social and economic issues - to fail implies fewer job opportunities or approval in examinations and higher costs, by repeating the school year or the subject (Cury, 2007, pp 1-2).*

Borasi (1996) brought important contributions, especially in the insertion of new objectives in the reform of School Mathematics in the United States, guiding teachers to use experimentation in the classroom and encouraging students to verbalize their ideas and arguments. Inspired by the ideas of Lakatos, Kuhn and Kline, Borasi found answers to important questions in the use of errors in research and teaching in Mathematics, proposing learning environments where the potential of errors can be used.

There are not many works with university students involving error analysis. We can cite the work of Guilhermo (1992) who classified the errors of students aged 14 to 20 in exercises involving properties and algebraic operations, finding errors that are repeated in other domains such as difficulties with notable products and the use of false rules to operate rational numbers. Like Borasi, Pochulu (2004) in his studies also concludes that correcting errors does not contribute to their elimination. Both adopted strategies to discuss errors with students in the classroom. Moreover, Milani (2002) analysed the results of meetings with a group of Calculus I students to understand the difficulties and conflicts encountered when these students deal with Calculus concepts according to the infinitesimal approach. We state that these methodologies have as a goal the development of mathematical resilience in students.

Mathematical resilience (MR) is defined as "a student's attitude toward mathematics that allows students to continue learning despite finding setbacks and challenges in their mathematics learning journey" (Johnston-Wilder & Lee, 2010, p. 38). It is known that all learning requires resilience to some degree, but the authors state that the resilience needed to learn mathematics (Mathematical Resilience) is:

> *a particular construct due to the difficulties presented when learning mathematics. This is partly because of the type of teaching that has often been used - tedious, isolationist, machine learning, elitist, and depersonalized (Nardi & Steward, 2003) and partly because of widespread beliefs about the fixed nature of mathematics skill (Johnston-Wilder & Lee, 2010, p. 38).*

There are four aspects to mathematical resilience which are dependent on: establishing a **growth mindset** so that students believe in themselves and believe that their mathematical skills can be developed with effort and hard work (**struggle**); believing that mathematics can be of great **value** for them and in society; getting to know how to work at learning mathematics; and getting to know how to find **support** to stay in the "Growth zone" (Lee & Johnston-Wilder, 2014).

According to Legault (2017), "Self-determination theory (SDT) is a broad theory of human personality and motivation concerned with how the individual interacts with and depends on the social environment". As we are interested in observing psychological aspects, one aspect of SDT that seems pertinent to this study is relatedness. Deci and Ryan (1985) established aspects of SDT which inculcate motivation, which are the basic psychological needs of **relatedness**, autonomy and competence. Relatedness requires experiencing others as responsive, sensitive, and caring, and in turn being able to be responsive, sensitive, and caring to others. Relatedness involves feeling connected and involved with others and having a sense of belonging (Ryan & Deci, 2017). Since then, research findings have emerged which include findings in mathematics education (Durmaz & Akkus, 2016; Lazarides & Rubach, 2017). In SDT it is assumed that people are proactive and not passive which means that the growth state is expected (Vansteenkist & Ryan, 2013). However, social, economic and cultural conditions may help or disturb the growth state (Ryan & Niemec, 2009). Therefore, where relatedness is not experienced, MR is threatened as learners may not find the support they need to be in the Growth Zone.

The main goals of this work were to develop an unprecedented activity using Information and Communication Technologies (ICT) to encourage reflections, presenting errors and working on ways to resolve any problems. The idea was to hold debates with students about the solutions they proposed, present challenges that include errors, and to outline strategies for understanding the concepts and procedures for solving the problem. We believe that this strategy helps to provide all the environmental aspects needed to develop mathematical resilience in students.

Methods

There are several ways to try to turn common mistakes made by students into a teaching tool. We suggest some interventions in the classroom, coordinated by the teacher, based on prior knowledge of common errors: providing reflections, presenting errors and working on ways to resolve the issues; holding debates with students about the solutions they propose, presenting games and challenges that include errors and, from these, outlining strategies for understanding the concepts and procedures for solving the problem. We observed that these activities require the teacher's experience and knowledge of the common mistakes that students make. Thus, part of this study is dedicated to sharing this teaching experience.

In the period of remote teaching due to the pandemic, there was a need to use ICT in education. An intervention case study of MR was carried out in vector calculus classes in which students had to, in a collaborative way, create questions and solve them. These students are on average 19 or 20 years old. The activity was applied to 160 students over one year (2 classes of 40 students per semester). The activity was performed in the Teams channel as if the work were conversations and had the following steps (see Figure 1): create a scalar function on R; create any curve in R^2 for a function previously defined by a colleague; solve the line integral for this curve and this function both given by two different colleagues; comment on a colleague's answer.

Results and Discussion

In Task 1, a common error was changing the nature of functions. Sometimes the students performed the intervention with their colleagues, politely saying that he or she had changed the type of function that the statement had requested, and the student could redo the step. In Task 3, the same occurred with the error in integral resolution in which there was the possibility of also redoing the activity.

The activity

Students had up to 2 days to make each task

Task 1 → Create a scalar function on R^2

Task 2 → Create any curve in R2 for a function defined by a colleague previously

Task 3 → Solve the line integral for this curve and this function given by two different colleagues

Task 4 → Commend on a colleague's answer

Figure 1 – The proposed activity diagram

This experience has been enriching, as it is an active methodology in which students analyse their own mistakes and collaborate with each other. The intervention happens naturally, and it is possible to see their growth as they embrace the content.

Some curiosities showed up during the interaction. The students did not use the time provided to them, as they answered the questions in much less time. This fact was surprising because they usually do the tasks in the last few minutes. During class, the teacher commented on this, and they said that they were anxious, and they wanted to do it soon. They also wanted to avoid the "most difficult" questions.

Examples of interaction

What does this dialogue reveal to us?

- *Kindness;*
- *There is the need of intervention;*
- *The student remakes his question*

There are two mistakes that end up changing the result. (The student described the mistakes) But the replacement in the integral is correct!

It's true, I'll correct!

- *It was recurring for them to thank each other*

There is a minus sign missing.

Thanks for the tip. I edited the answer.

Figure 2 – Examples of interaction during the experience

Other types of interaction

The students were willing to interact with each other and with me, and to collaborate respectfully.

This is linked to relatedness, which is one of the pillars of Mathematical Resilience.

"kkkkkkkk" that it's a way to simulate laughter in the internet language;

"tmj" that means "we are together"

"I'm going to face this question!"

Figure 3 – Examples of shared feelings during the experience

In Figure 2, examples of interaction can be seen. Students felt comfortable to share feelings of kindness and respect in the dialogues. They worked collaboratively with their peers, and they felt part of a community, as supporting each other made them feel less isolated. They could also develop the affective dimension of growth, that is, the confidence that all people can develop mathematical skill and can learn more mathematics with effort and support (from peers, ICT, internet).

Lack of relatedness can be a major source of mathematics anxiety, and MR has consistently stressed the nature of acceptance and support in a community of learners of maths (Johnston-Wilder & Mackrell, 2020). The interaction between students in this experience, as in Figure 3, enabled the construction of a community of learners of maths and the learning of how to recruit support to grow mathematical knowledge, understanding and competence.

Conclusions

The activity was innovative because it generated unprecedented questions since the students created new questions that are not found in literature. The students were the authors of the questions, which contributes to the growth of creativity and autonomy. In this way, they can become confident and better understand the mathematical concepts involved.

The created environment allowed for relaxing, joking and students could correct themselves without embarrassment. Therefore, the activity is upheld by the pillars of growth and relatedness. It is known how important interpersonal relationships between students are to create an environment conducive to learning. Hence, we hope that more activities like these can be developed. As a future question, we propose a similar activity in the face-to-face classes and will analyse the results, applying a questionnaire at the end of the semester to know the student's opinion about such activities.

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Paper 6

Mathematics Anxiety – some ideas about its origins

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Abstract

Literature about mathematics anxiety (MA) is well established, however there seems to be a lack of agreement about its origins. MA affects many people to the extent that they actively avoid mathematics and others find they cannot engage in any mathematical activity. Knowing more about the origins of mathematics anxiety seems useful. This paper analyses the narrations of the prior experience of a group of adult learners who were assessed as mathematics anxious. Using narrative research techniques, their stories were analysed, so that their voices could be heard. They tell a story of negative experiences in learning mathematics, many attributable to the way their teachers considered it appropriate to construct their learning environment. Those who did not readily respond to teaching were ignored, humiliated or actively removed from the class. Teachers did not offer support, to the extent that the participants questioned: did they not know how to offer the support needed?

Keywords: adult learners, anxiety producing pedagogies, teaching for resilience.

1. Introduction

Meera Syal spoke on Radio 2 in 2021 about her teacher, a Mr Robertson, who had so believed in her and inspired her that she credited him with enabling her to become a successful actress, writer and comedian. This paper is not about Mr Robertson. Although it is important here to acknowledge that there are many "Mr Robertson's who build relationships with their students that foster independence and self-confidence. Both authors of this paper were mathematics teachers, and we still work with teachers and perhaps more importantly, respect teachers and believe that the job they do is both difficult and demanding, and that they do it because they want the best for the learners. We have to state this because what we are going to report sounds like an indictment of teachers, particularly mathematics teachers. It is not. What it is, is a study into the origins of their mathematics anxiety as remembered by the adult learners in this study. It is their voice that is presented here, and the narrative methodology used was designed to allow that voice to be heard.

For a while now, both of us have been studying and researching the effects of mathematical anxiety on students and have worked with the ideas that are termed mathematical resilience (Lee and Johnston-Wilder, 2010). As we have engaged in conversations it has become clear that adults, who acknowledge their own mathematics anxiety, remember significant, often traumatic, events to which they attribute the start of that anxiety. This paper presents the words of a group of mathematically anxious adult learners, aiming to give voice to the truth as they see it.

2. Methodology

"Narrative research is not only the stories or accounts contributed by the participant, it is the evaluating and analysing those accounts" (Overcash, 2003, p.108). Here we present accounts of participants' experiences of learning mathematics in school, and evaluate and analyse those accounts to understand the origins of mathematics anxiety. Moen (2006) attests that, for most people, storytelling is a way to create order out of experience, whether traumatic or positive. Focus groups were used within this study to allow the stories to be told. The narratives that the participants told convey truths of their lived experience. John, who had taught the participants, conducted the focus groups. These adult learners were taking mathematics as part of an access course, enabling them to qualify for university. A total of nine focus groups were convened with between 4 - 6 students in each group. The recordings of the focus groups were transcribed verbatim prior to analysis. Each focus group were asked similar questions following a semi-structured schedule.

3. Findings

The data analysed for this paper are transcribed utterances, freely contributed by the participants, with full ethical permissions, to explain why they felt there had been barriers which had prevented them from learning mathematics during their time in school, either primary or secondary school. Analysing these data using reflexive thematic analysis (Braun and Clarke, 2019) yielded six themes that the participants' narrations fell into:

You can run but you can't hide: The data revealed students who would have liked to avoid mathematics completely, but they now realise that they need to engage with mathematics if they are to fulfil their life-plans. S1 said "*When I was in school, I really couldn't wait to get away from the maths lessons I felt that stressed about being in there*" which was backed up by S4 "*I really am the type where if I see some stress coming towards me, I just want to run I want to get out of that situation*" and S3 "*I feel that I need all this to get into my social work qualification and I feel there's nowhere to hide …. This gives me a motivation now to achieve, which I didn't really have when I was in school.*"

The innate resilience of these students, now they are adults, is evident in these statements. Despite the remembered lack of motivation and stress of being in a mathematics lesson in school, they recognise that they would like to continue to hide but they cannot.

Don't put us on the spot: This theme shows the depth of feeling experienced by learners when they were singled out to provide answers in a mathematics lesson. Being put "on the spot" is the participants' way of expressing being publicly singled out to give an answer in mathematics. This common pedagogical choice seems to have been remembered as damaging the teacher-pupil relationship more than any other. The damage appears to have been caused by feeling of humiliation and of inescapable stress being caused.

> *S11: "The Teacher had me up in front of the class knowing full well that I didn't understand it and that I had repeatedly asked for help, she got me to do the sum she knew I couldn't do in front of everybody, and I was ridiculed, it was awful."*

The participants commonly used strongly emotional expressions such as "hate" or "killing" in discussing their memories of this teacher action. S5 said "*I hate being put on the spot in maths.*" and S2 seemed to feel similarly saying, "*I just have felt really bad when I was put on the spot, and I felt like killing him.*"

The ruptured teacher-pupil relationship is further elucidated by the lack of respect shown to the teachers who put them "on the spot".

> *S2: "When my teacher would ask me questions, I would just look at him as if he was a 'prat' and just ignore him … I really didn't like being put on the spot, so that was just the outcome, and it wasn't very good really."*

The participants expressed feelings of incompetence and lack of control, which resulted in many of them suffering anxiety that amounted to being traumatised. The pedagogical device of singling out pupils and asking them to quickly answer questions engenders strong emotions.

Just one incident: The participants would point to one particular incident as the time that they began to dislike being part of mathematics lessons, and want to take steps, such as avoidance, to safeguard themselves from damage to their well-being. Some of the incidents the participants recall happening seem surprising to us, especially since most of the participants were at school about ten to fifteen years ago. However, this is what they told us happened.

> *S12: "Because I didn't understand, I got upset and they kicked me out. A horrible experience and I have hated Maths ever since. … I don't think Maths really bothered me before that point, and after that point it stopped me from doing well."*

> *S4: "Stand in front of the class and make you do something they know you can't do so everyone could laugh at you. Honestly there was nothing I liked about Maths."*

One incident, if it is humiliating or emotionally upsetting, appears to be able to colour a learner's attitude to mathematical learning for the rest of their learning career and severely affect their life choices.

Don't throw me out: Many students reported a lack of access to any support to help them understand mathematics. For some, asking for help seemed to prompt the teacher to remove them from the classroom.

> *S12: "My Teacher used to chuck me out all the time. I didn't find they were as supportive. They would just give you the work and say just get on with it."*

> *S4: "Our teacher used to chuck a chalkboard rubber at you, yes, they had a rubber, and they would chuck it at you. … they never tried to learn us, the ones that were stupid were put in a corner. Yes, they never tried, they just used to put me in a corner and then that was it."*

> *S5: "They just don't explain, they go through it and then just don't help you. You are just expected to do it."*

That these learners could not access support would further convince them that they had no place in the community of those that learned mathematics. They were excluded from some notional elite that had a right to learn mathematics. The participants who reported being thrown out of class, being told to colour in pictures in a corner or who were otherwise excluded from the learners in the classroom, were, considerably later in their lives, back in the classroom, learning successfully. They would have preferred to have received the support they needed when in school.

Teachers need more knowledge of how to help: The participants were adults, and as such maybe they were willing to look for reasons as to why their teachers were unable to help them to understand and make progress with mathematics. One attributed their teacher's lack of support to a lack of experience: S7: "*He could see that I was struggling, and you could see that he just didn't know what to do, he wasn't experienced enough to help me.*"

However, some participants felt that it was their fault that the teacher was unable to help.

> *S8: "It's not all about the teacher being at fault … I had a mental block, and I just knew I wasn't going to be able to do this." One of the participants advised:*

> *S10: "Could the maths teaching be presented using different techniques so that we could do the same problem but in different ways? So, if you could see a student that wasn't getting it when you were teaching one way, then you would switch to another method so that would enable them to be able to understand it that way."*

Teachers require the experience to understand that mathematics anxiety may cause students to feel that they have a mental block, but that with the right support such feelings can be overcome. All learners must be assured that they have the right to the support they need, and to be encouraged to ask questions until they do understand. This is about relationships. Where the relationship is one where the teacher is truly interested in everyone succeeding then the whole class can engage in a joint endeavour to assure that everyone succeeds.

Is it the teachers' fault? The participants in this study were adults, and perhaps one of the most convincing aspects of their reports is that they wanted to talk through the extent to which they felt they should hold their teachers responsible for the trauma they reported. The data in this theme shows contradictory views but also shows the thoughtfulness of the participants. Some held their teachers responsible; when asked "*was it your teacher's fault*" one commented:

> *S12: "Yes I think so definitely. I don't think they intend to do it, but I think the more naturally able ones just tend to have that better relationship with the maths teacher because they understand it. I do get that teachers must get frustrated with people that don't try but if you are trying they sometimes, because you can't do it and are trying your best, they kind of push you aside with the rest of the students."*

Others agreed, one student said "*And when we thought of Maths, we thought oh Maths! Like the class itself was horrible. I always felt that I could understand it but then the vibe of the class always made people feel they couldn't do it.*" (S8) But there were disagreements, several felt that any blame for the relationship not working was to be shared jointly "*I would say it's equal blame on both, because you know the teachers, a lot of time in school the classes are overcrowded so it's a lot for the Teacher. They are trying their best to separate the ones that will need the extra help and the ones who can do it naturally but it's not always going to work. It's the attitude in them, so I think it's both equally to blame*" (S3).

The maturity of the participants is evident in these quotes, and they raise pertinent points, discussing ideas that are prevalent in society. The idea that some people can "naturally" do mathematics, and some cannot, has been discredited by research such as Yeager and Dweck, (2012). However, when talking about previous experiences, they accept that teachers would naturally form good relationships with those who they felt were inherently able to learn mathematics, and that those that struggled were less interesting to the teacher.

4. Discussion and Conclusions

The participants have clear messages about what it takes to build a mathematically resilient and literate society. This requires every mathematics teacher to be, "*A competent teacher [which means] you enjoy the lessons, [and] they make you feel at ease*" (S6). There are several ideas here for accomplishing this classroom environment, some are about what it means to be a competent teacher and others are about the pedagogical choices that mathematical teachers make.

A competent teacher, according to this data, considers the relationships within the learning environment and ensures the right support is available. In a classroom of 30 or more students, support is more quickly obtained if the teacher activates the students to support one another (Wiliam, 2017). This is something that adult learners do as a matter of course, but research (Lee and Johnston-Wilder, 2013; Nardi and Steward, 2003) has made clear is often not conventionally allowed in a secondary mathematics classroom.

Teachers should be wary about putting students "on the spot." Singling out students to answer questions can lead to feelings that they are being laughed at, and consequent humiliation. Ashcraft and Krause (2007) are clear about the link between this pedagogical choice and the development of mathematics anxiety. A teachers' own experience of school, first as a pupil, then as a teacher, will have an influence on their teaching alongside any formal higher educational experience of mathematics. Meirink, et.al., (2009) found teacher beliefs had a significant influence on their classroom practice and are relatively difficult to change. Teachers who have experienced mathematics taught in the way experienced by these participants are likely to continue the practice.

At the time the participants were in school, their teachers did not use strategies that would help them close the gaps in their understanding. If their teachers had no ideas of how to help, they may well blame the student rather than face their own lack of knowledge. The reactions reported may have been a way of the teachers safeguarding themselves. In these times of Covid this aspect is particularly pertinent. Many learners have been absent from school or have suffered through teacher absences; almost all will have significant gaps in their knowledge and understanding of mathematics. If mathematics teachers still do not know how to respond and support their students to close those gaps, then many more students will not develop the mathematical understanding they need to flourish in the future.

Whilst we agree with the participants in not attributing all the blame to the teachers, we would be less willing to blame the students who were just teenagers. These young people had been placed in attainment groupings that limited both the teaching experiences they received and the expectations of success from their teachers (Francis, et.al., 2020). They were almost certainly given the least experienced or least qualified teachers. It seems likely that they lacked motivation, but not for any intrinsic reason.

The participants were looking for a relationship with their teacher that supported their learning, that made them feel competent and in control of their learning. A relationship where they could safeguard their well-being. Instead, they reported being ignored, humiliated and sometimes "thrown out" of the classroom. They responded by "hating" both their teacher and the mathematical learning. They also developed such anxiety that they "dreaded" going to lessons. The teacher-pupil relationship is key to effective teaching, "Curricula, lesson plans and learning outcomes are long forgotten, but the impact of relationships lives on" (Giles et al., p. 214). Witmer (2005) saw relationships as the building blocks of effective teaching and student success.

Is it more difficult to build that effective relationship when the students are involved in learning mathematics? It may be. Nardi and Steward's (2003) research shows experiencing the traditional way of teaching mathematics makes it difficult for most students to feel involved or that mathematics is worth the effort it takes to learn. The UK system adds to the difficulties of learning mathematics by, for example, demanding answers to mental maths questions within

five seconds on the KS2 Mental Maths National Assessment Papers, when research has shown clearly that demands for speed can mitigate attainment in mathematics and cause anxiety.

This paper makes clear the need for building positive relationships. The students' relationship to mathematics is likely to have been damaged by systematic demands on the way that mathematics teachers view good practice, but it can be repaired. The literature on mathematical resilience (such as Johnston-Wilder et al., 2021) has much to say on this point. It is very clear from the data that John and Alex, the teachers of these groups, are repairing students' relationship with mathematics. One student (S12) stated that her career aspirations were 'thwarted' because "*my maths wasn't good enough for me to get into university and I mistakenly thought if you can't do maths – you're not smart*". She now understands her mistake and knows herself to be smart enough to achieve her aspirations.

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Paper 7

Recognising and Coping with Mathematics Anxiety in Adult Learners

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Abstract

Many mature Access to Higher Education (HE) in Health students returning to education find mathematics to be difficult and the cause of some anxiety. This article reports on a research programme undertaken into interventions designed to help a group of these mature students more easily engage with mathematics. The research investigated whether, and to what extent, mathematical anxiety existed in the students using the Mathematics Anxiety Scale developed by Betz in 1978. Following this, an intervention was used to help the students understand why they might feel anxious and to introduce strategies to overcome such feelings.

The research revealed significant numbers of students (81%) did have mathematics anxiety. However, after a short presentation and discussion, the participants self-reported lower levels of anxiety and a greater tendency towards a fixed mindset, suggesting that it is possible to reduce anxiety levels in mathematics. This paper discusses the outcomes of a pilot study in 2019.

Keywords: adult learners, successful intervention, growth mindset, improvement.

1. Introduction

For many years it has been noted that mature, returning learners can often have difficulties with mathematics as a part of their course of study Johnston-Wilder et.al., 2013). Whenever mathematics is part of the course, there is a group of students whose anxiety and fear seems to increase as a result. Researchers have found that many people have negative attitudes towards mathematics, which in some people is described as "a "general fear of contact with mathematics" (Hembree, 1990, p.45). This fear can inhibit learning; many people find mathematical tasks difficult, to the point that they exhibit phobia or anxiety, or at least avoidance from engaging in any endeavour that could require mathematical reasoning (Johnston-Wilder and Lee, 2010, p.1). Thus many students are unable to progress in their chosen field as they cannot face mathematics.

The research discussed in this paper was part of a larger project exploring how adult learners can be enabled to learn mathematics both more willingly and more effectively (Morgan In press). The students were part of an access to higher education course which if they succeeded would allow them to continue to study health care. The data reported here is from two questionnaires, one taken before an intervention and one at the end and from interviews that were held after the intervention. The intervention consisted of a presentation detailing the extent of mathematics anxiety that is experienced nationally along with ideas about what can be done to develop the resilience needed to overcome the barriers that learning mathematics can present.

2. Mathematics Anxiety

Mathematics anxiety as a phenomenon has been studied for over 60 years (Dowker et al., 2016). It can be defined as "feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in ordinary life and academic situations" (Richardson & Suinn, 1972, p.551). One recurring theme in the mathematics anxiety literature is unpleasant outcomes; learners have reported feeling sick when faced with a problem as well as various other negative feelings (see for example Hembree, 1990; Ashcraft, 2002). Lewis (1970) defines this anxiety as an emotional state, with the subject experiencing fear and dread. He states that the emotion of anxiety is unpleasant and is out of all proportion to the threat the student imagines. Thus, if students perceive a constant threat their progress will be impeded.

Even within seemingly technical vocations, where mathematics is to be expected, anxiety is present. Dowker et al. (2016) cites Johnston-Wilder and Lee et al. (2014) who found that "about 30% of a group of apprentices showed high mathematics anxiety, with a further 18% affected to a lesser degree" (Dowker et al., 2016, p.3).

2.1 What about anxiety in adult learners?

Much of the mathematics anxiety literature concentrates on school age learners but recognises that the barriers to learning mathematics seen in young people continue into adulthood. Dowker et al. (2016) note that negative attitudes towards mathematics (and mathematics anxiety) seem to increase as children reach secondary school age and that this increasing negativity continues into post-secondary education and throughout adulthood. By examining mathematics anxiety and its possible links with mathematics avoidance and perceived failure it could be shown that targeted interventions may help to present mathematics in a way that reduces the anxiety, and which may improve success rates for mature students. The aim of this short research paper, which is a part of a much larger PhD research project, was to contribute to furthering the knowledge and practice in the field of mathematics learning for students in a mature age group.

Recent research by Carey et al. (2019) affirms that the UK could be going through a 'mathematics crisis'. The evidence suggests that

> *"functional literacy skills amongst working-age adults are steadily increasing but the proportion of adults with functional mathematics skills equivalent to a GCSE grade C has dropped from 26% in 2003 to only 22% in 2011. This number is strikingly low compared with the 57% who achieved the equivalent in functional literacy skills" (Carey et al., 2019, p.6).*

These statistics have implications for this research because the target group is adult students who seek and aspire to professional roles in health and social care where functional numeracy (and evidence of it) is a requirement.

2.2 Affective Issues

Hoyles (1975) explained that affective issues are often a topic of discussion amongst students in the mathematics classroom, i.e. whether they like or dislike mathematics or whether they find the concepts presented difficult or easy. Students, and others, often make statements such as 'I hate maths', 'I will never understand maths' etc. evidencing that students often have strong feelings towards mathematics, and they are often negative. The affective domain concerns how students feel towards mathematics as a subject. Roth and Walshaw (2019) stated that the affective domain includes ideas such as beliefs, attitudes, engagement, motivation, and anxiety.

3. Data Collection

To promote validity, this research was carried out with an awareness of potential sources of bias throughout the processes of design, data collection and data analysis. For instance, Dodou and de Winter (2014) affirm that social desirability bias may induce respondents to respond to questions in a way that will lead them to being accepted and liked. This phenomenon has been limited by focusing on unconditional positive regard whereby I informed the students that it was OK to respond in any way that seems correct to them regardless of any thoughts of social desirability.

I also used caution regarding researcher bias, which happens when a researcher seeks confirmation of beliefs they already hold; here this might have taken the form of inferring anxiety where none exists. I aimed to avoid leading questions and maintain an awareness of the halo effect where the student may be viewed in a certain light based purely on the answer to one question or positive attribute. Such bias can be mitigated by firstly recognising that such bias may exist and then remaining aware of this throughout the process of design, data collection and data analysis.

The data collection occurred in two stages. First, each participant completed the Mathematics Anxiety Scale (MAS). The students then took part in short, follow-up interviews to explore how the students were feeling towards anxiety and the effects of the intervention. This allowed data to be gathered that evidenced the effects of the interventions and to promote the validity of the research

3.1 Surveys (MAS)

The Mathematics Anxiety Scale (MAS) seemed to give an acceptable internal consistency and test/retest reliability, making it suitable for replication (Pajares & Urdan, 1996). Consistency is essential if the research is to be trustworthy, that is if others are to be convinced that the data does reflect the feelings of the student groups being studied. I used the shorter 10-item Mathematics Anxiety Scale (MAS; Betz, 1978) to measure anxiety in the cohort, as it had already been used successfully in the UK to measure anxiety in apprentices (Johnston-Wilder et al., 2013) and would not take the students too long to complete.

4. The development of an intervention

For Uusimaki and Kidman (2004), the first step in solving the mathematics anxiety problem is to raise student awareness that mathematics anxiety exists and that it can be fixed. For the intervention reported on in this paper The anxiety presentation was developed in order to show students that mathematics anxiety exists and that it may be minimised to allow students to succeed in mathematics.

4.1 Intervention: The Anxiety Presentation

To design an effective intervention to assist the Access to HE students in reducing their anxiety, the work of Johnston-Wilder and Lee was considered. As a result of their on-going research Johnston-Wilder and Lee (2017) developed five characteristics of a learning environment that is known to build mathematical resilience, it should:

1. help students know that brain capacity can be grown.
2. enable everyone to feel included in and supported.
3. help students to see mathematics as relevant in the world in which they live.
4. ask students to struggle, but not too much.
5. model ways to work at maths, showing how to get appropriate support

In the presentation it was made clear to the students how widespread mathematics anxiety is and that they should not feel alone if they also suffered from it. It was also emphasised that they could challenge and defeat negative feelings if they were prepared to accept support in doing so. Dweck (2007) asserts that if students realise that mathematics anxiety is 'normal' and widely accepted as a debilitating state then they will be that much more able to challenge and defeat their negative feelings in pursuit of mathematical proficiency. To ascertain if anxiety was measurably reduced following the presentation, a 'Before and After' poll was used to measure the mathematical anxiety of those listening to the presentation.

5. Results

The MAS surveys were used in October 2019 with a group of 77 Access to HE Health & Social Care students. During informal discussions with these students, they had revealed their feelings towards mathematics using such terms as: *terrified, dreadful, scared, cripplingly anxious and dislike*. Literature implies (see for example Ashcraft, 2002, and Cates & Rhymer, 2003) that mathematics anxiety would be likely to cause underachievement at mathematics. The students in this cohort had not succeeded with mathematics in school and therefore it seemed likely that anxiety may be the cause. The first questionnaire tested if this was the case.

5.1 The MAS Results

The first surveys had a response rate of approximately 79% out of the whole cohort. As the results show (see Figure 1 below), a large proportion of the students within the group were showing significant signs of mathematics anxiety. Overall, the survey revealed a total of 81% of all students who responded experiencing some form of anxiety towards learning mathematics.

% of Group	MAS Scores	No. of Students	Range of Anxiety Levels
68%	32 and above	52	Visibly anxious
13%	Between 27 and 31	10	Anxious but not visibly
19%	Less than 27	15	Not measurably anxious

Figure 1. MAS Scores for October 2019

5.2 The before and after results of the presentation

The results of the final survey seem to show that with the focused interventions, some of the students' anxieties can be removed or reduced helping them to re-engage with mathematics as a subject.

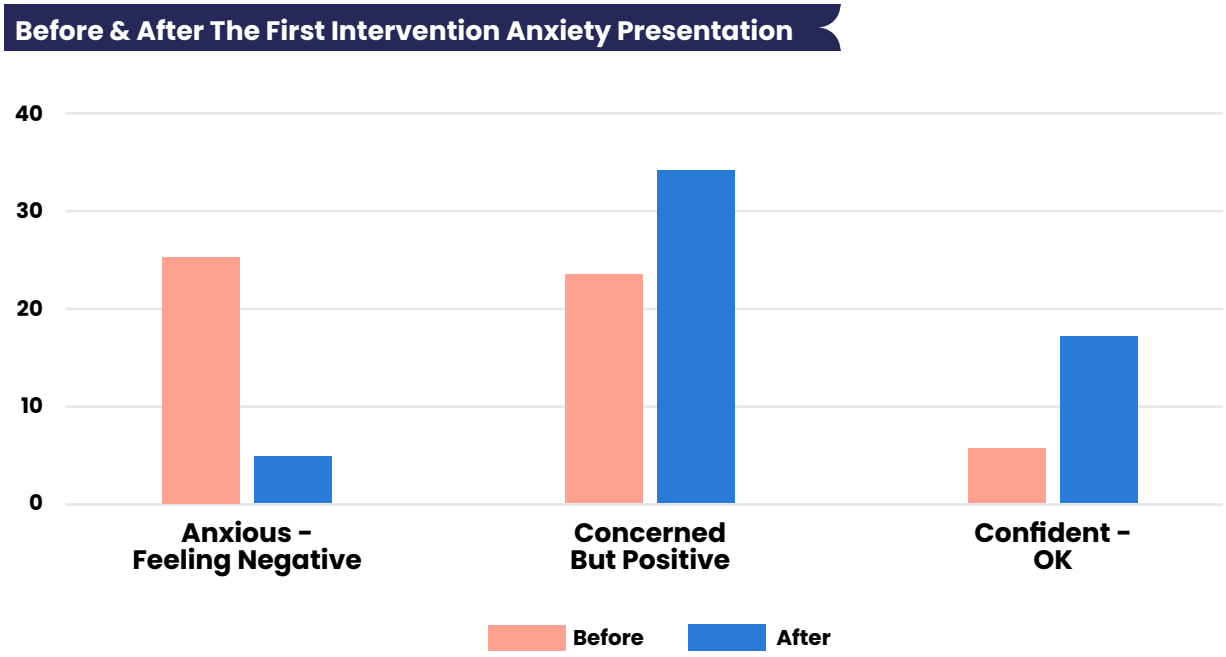

Figure 2. Feelings before and after the first intervention

In addition to the survey responses, I was also able to collect other data in interviews. At the conclusion of the presentation, I took the opportunity of asking students how they felt about the intervention. The students made a number of statements which included:

> *"I didn't realise that there were so many people in the world who felt the same as me about mathematics".*

> *"I thought it was just me".*

> *"Now that I know I'm not on my own, I feel that I will be able to give mathematics a much better try than I did in school".*

> *"When I attended school I felt isolated and alone, now I feel much better placed to do this and win".*

> *"I felt brushed aside when I was in school, I couldn't do mathematics, my teacher knew it and he just ignored me and concentrated on the students he thought could do mathematics".*

Reviewing the data from the surveys with the short interviews after the intervention talking to the students about mathematics anxiety appears to have helped them considerably.

6. Conclusions

This research was approached with some caution; there was concern about how students may respond to such an emotive topic. Many students who have a negative experience very often want to distance themselves from anything to do with that experience. Fortunately, this has not been the case with this target group. In most all cases they have been very positive about the scheme and have willingly participated. To an extent the biases and limitations which are an inherent part of this type of research do not allow for generalisation, but the data does suggest that mathematics anxiety can be reduced in adult returning learners.

The existing literature has repeatedly demonstrated that mathematics anxiety exists and can cause major disruptions to students' mathematical learning, if indeed they try to pursue their learning at all. From the data reported above, 81% of those surveyed suffered with mathematics anxiety, either visibly or well disguised. However, this research has shown that an appropriate intervention, can help decrease mathematics anxiety in the short term. However, to enable learners to break the cycle of failure in mathematics and begin to progress towards their desired qualifications and career goals, other interventions will be needed. These results present a promising start

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Paper 8

Teaching Newton's Binomial using an ICT environment and a STEM approach - a strategy to enhance mathematical resilience

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Abstract

During the COVID-19 pandemic, technology was crucial to make mathematics education possible. Combinatorial Analysis in Basic and Higher Education has proved to be a great challenge. This work aims to create a Didactic Sequence (DS) using the ICT environment to enhance Mathematical Resilience. In designing the DS, the complex ideas of Newton's Binomial could be made more understandable by using them to the concept of genetics. In that way a STEM approach was taken by applying mathematics to science. The DS is divided into two parts which are summarized in two distinct and complementary activities, both produced through Google Forms. They were designed in a student-centered learning environment, with more active participation, supporting the idea that mathematics is a valuable subject and is worth studying. We analyzed notes from Google Forms and found that this study brought greater access to Combinatory concepts and increased engagement and interest in the learning of mathematics. In conclusion, the DS shows the potential that this type of intervention can contribute to strengthening mathematical resilience among learners.

Keywords: Mathematical Resilience; Didactic Sequence; STEM; Combinatorics.

Introduction

COVID-19 has drastically transformed the teaching and learning process, it forced the educational system to shift from traditional face-to-face learning to online learning. During this period, technology was crucial to making mathematics education possible as it is an essential tool for social communication and global competitiveness.

Mathematics Anxiety (MA) and lack of confidence, however, have to be considered in this scenario and have consequences such as underachievement in mathematics, especially when there is a great need for mathematical thinking as the world confronts challenges related to climate change and social and economic injustice. Underachievement in mathematics is widespread and may be linked to Mathematics Anxiety (OECD, 2013) which is particular troubling in contexts where STEM proficiency has a great demand (Correia et al., 2010). Combinatorial Analysis, in particular in Basic and Higher Education, has proved to be a great challenge, both from the point of view of teaching and learning. This subject tends to be explored in a mechanized way, with the memorization of formulas and calculations that do not contribute to the construction of combinatorial reasoning (Sabo, 2007).

This work aims to create a Didactic Sequence (DS) using an Information and Communication Technologies (ICTs) environment to enhance Mathematical Resilience (MR), which is defined as "a learner's stance towards mathematics that enables pupils to continue learning despite finding setbacks and challenges in their mathematical learning journey" (Johnston-Wilder, 2018). The activity is interdisciplinary and contextualized. The DS took a STEM approach (Science,

Technology, Engineering and Mathematics), to present the content of Newton's Binomial in line with the concepts of Genetics.

Didactic Sequence (DS) using ICT

For this work, we developed a Didactic Sequence (DS) in the pedagogical scope, which is defined by Zabala (1998, p.18) as: "a set of ordered activities, structured and articulated for the achievement of certain educational objectives, which have a beginning and an end known by teachers and students". As mentioned previously, the DS was intended to aid students learn about Newton's Binomial. According to Avakian (2016), education focused on STEM "helps to identify skills and knowledge, to activate scientific and mathematical thinking as [learners] gain skills practices, and to sustain interest in science, mathematics, and technology in the future".

The DS is divided into two phases, which are summarized in two distinct and complementary activities, both produced with Google Forms, called "Expanding Genetics" and "Binomial's fault". The activities were guided by an interdisciplinary perspective and a playful environment, relating Genetics and Newton's Binomial concepts. This DS intended: to awaken the student's interest in mathematics and its application in genetics; to contribute to the transformation of the school environment into a space for creative learning using the ICT; to support learners in trying out solutions to problems proposed in each animation; to assist learners in making the transposition from concrete to abstract, and to aid learners to identify and understand the development of Newton's Binomial and its expansion.

Expanding Genetics

The first form contains questions, GIFs (Graphics Interchange Format), and videos to supporting learners in building their knowledge about genetics. It begins with a video about Combinatorics, some concepts of Genetics, and the history of Mendel. In this experience, we let the learners try many possibilities in a non-supervised environment that encourages them to try different solutions – promoting learners' autonomy – so that they can develop a mindset for MR. The form finishes with a video about what was intended to be learned and a final challenge.

By changing the usual teaching methodology, we sought to foster creative, investigative thinking and problem-solving. We did not want learners to memorize formulas but to learn to reflect on how formulas are formed and how they are expanded using a real-life application of the ideas.

Binomial's fault

The second form has three pages: Identification, Questions, and Conclusion. It intends to verify more formally, the student's learning, through four questions about the content presented in the first part of the DS. Both activities were designed to create a student-centered learning environment, with more active participation, supporting the idea that mathematics is a valuable subject that is worth studying.

Mathematical Resilience Ideas used to develop the DS

Mathematical Resilience (MR) is defined as "a learner's stance towards mathematics that enables pupils to continue learning despite finding setbacks and challenges in their mathematical learning journey" (Johnston-Wilder & Lee, 2010, p.38). It is known that all learning requires resilience to an extent, but the authors claim that the resilience required for learning mathematics (Mathematical Resilience) is "a particular construct because of the difficulties presented when

learning mathematics" (Johnston-Wilder & Lee, 2010, p. 38). This is, in part, because prevalent mathematics pedagogy often leaves learners feeling that learning mathematics is tedious, isolated, rote, elitist, and depersonalized (Nardi & Steward, 2003) and in part because of pervasive fixed mindsets (Dweck, 2000) in relation to mathematics.

The DS was designed with the idea of developing MR in the students. Some MR ideas used to develop the didactic sequence were playfulness, interdisciplinarity, value, and autonomy. Next, we describe each of them linked to the DS.

Playfulness

The simulations presented in the 'Expanding Genetics' form are playful, and they facilitate checking mentally the possibilities for each question. In Figure 1, learners verify the GIF animation and the transformation of the seeds into letters. The learning environments can elicit joyful sensemaking experiences that lead to a rich understanding of STEM concepts according to Sengupta-Irving and Enyedy (2015). Also, in this perspective, playfulness is important in designing and creating spaces that value learners' agency and affect (Horn, 2017; Jaber & Hammer, 2016; Keifert et al., 2017).

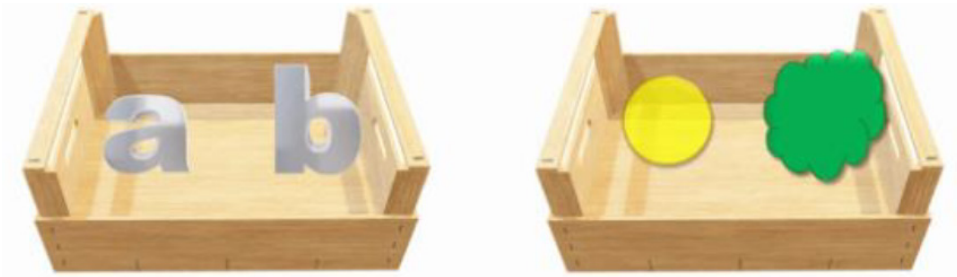

Figure 1 - Transformation of seeds in letters.

Interdisciplinarity

Interdisciplinarity aids learners to perceive and make meaningful their learned mathematical skills, supporting them to use mathematics as a tool to reach various goals and solve problems in daily life. With the emergence of connections between disciplines, learners' interest in mathematics increases (Hathout, 2016). In addition, building newly learned knowledge on connections will turn mathematics into a challenging and exciting learning experience (Hathout, 2016). Interdisciplinarity connects facts from the learner's personal, social and cultural life, illustrating Vygotsky's (1978) theory about social interaction through which the individual learns and builds his learning. Vygotsky (1978) argues that interactions affect the development of higher order mental functions such as formal reasoning. In this sense, the activity provides these interactions, since it enables dialogues with objects and simulations in an interdisciplinary context, thereby enabling knowledge construction.

Value

According to Chouinard & Roy (2008), learners will be more interested and more motivated to study mathematics if they believe it is valuable. The first form was planned to develop this sense of value and purpose in relation to mathematics. It related Genetics (an application from the real world) with Newton's Binomial concepts. It is the case of quantitative or polygenic inheritance, where two or more pairs of genes participate. The interaction that occurs between the genes (polygenes) that transmit the inherited traits happens in such a way that each one of them is responsible for a portion of the resulting phenotype. The inheritance distribution pattern, in this case, follows the Newton Binomial pattern, where $(p + q)^n$, where n is the number of polygenes. Examples of quantitative inheritance include characteristics such as skin color, human eye color,

height, weight, and hair color. The construction of knowledge expresses the activity of learners as they give meaning to reality; the interactions and the way they are established collaborate significantly in these constructed senses, re-signifying them in a continuous process (Piaget & Garcia, 1987).

Autonomy

Making available tasks that are interesting or important to learners, and which foster the idea of the value of mathematics, supports the development of autonomous motivation in learners according to Stroet et al. (2013). Intellectual autonomy has been defined as, "students' awareness of and willingness to draw on their own intellectual capabilities when making mathematical decisions and judgments" (Cobb & Yackel, 1998, p. 170). The activity encourages mental reasoning and learners' decisions. That these occurred can be verified in the following experience reports.

Experience reports

Experience report 1

First, we tested the activity on a group of students at the undergraduate level from a public university in Rio de Janeiro. The students reported that the experience was helpful in the fixation of content as the simulations and questions about the same inquiry, on different levels, assisted them in transposing from 'concrete' to 'abstract'. Thus, the activity encouraged the construction of knowledge, and the students related that the situations and problems were more fun to solve. They also reported that the problems where everyday life meets the mathematical world and not fictional ones were less difficult to understand and solve. Finally, they related that the activity turned out to be more motivating due to the link with Biology.

Experience report 2

Secondly, we tested the activity on a group of students from a high school in Rio de Janeiro (15-17 years old). A Brainstorm activity was performed before the intervention with the group using Google Meet. The students reported their feelings towards the written word MATHEMATICS which included: *problem, pain, test, hatred, fear, sadness, and anguish.* Their opinion/feelings about the activity were collected from the form and included statements such as: "researching new content was motivating"; "learning with entertainment was less boring to learn"; "the activity was dynamic and that contributes to better learning"; "the power of learning better gives me more opportunities in life".

Conclusions

This brief small-scale study brought learners greater access to Combinatory concepts and seems to have increased engagement and interest in the learning of mathematics. The feedback received from the learners shows that there is potential for this type of intervention to build MR. In future works, we plan to investigate the use of playful and creative activities inserted in the STEAM approach, with "Arts" included, (games, video production, photography, and interdisciplinary projects) as a way to help student understand and overcome the ever-present barrier of anxiety towards mathematics and to support the development of MR in high school and undergraduate students. In addition, we plan to apply the ideas in public schools in Rio de Janeiro from FAETEC-RJ Network as part of our extension project from Fluminense Federal University - The Brazilian Antenna of Mathematics.

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Paper 9

New ways of communicating ideas about mathematical resilience to parents and carers

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Abstract

Parental engagement in general is beneficial, but parents with mathematics anxiety who help their children with mathematics at home can have negative impact on progress. However, parental mathematics anxiety can be overcome, leading to a positive experience for both parent and child.

In 2020, during lockdown, parents were called upon to support their children's mathematical learning at home. Wright, a Senior Education Improvement Adviser for Solihull, received reports that many parents found supporting mathematics very challenging. She therefore developed ways of offering support to parents.

Since 2018, Russell has given author talks to parents in libraries about how to help give your child a 'can-do' attitude to mathematics. When Covid-19 pandemic restrictions prevented this, she gave online talks to parents about her latest book, focusing on communicating mathematical resilience ideas to parents.

Here we report on new ways of communicating ideas about mathematical resilience, which have been positively received by parents.

Keywords: Parental engagement; learning mathematics; mathematical resilience.

Russell's Background

Over the years, I have taught mathematics in many school settings, lectured in a College of Further Education, coached and tutored students of all ages. However, I took a career break from teaching when my children were younger, and I ran a small fashion business. Suddenly I was not a teacher. I chatted a lot to bewildered parents (care givers, grandparents) who shared things they would not have said to me as a teacher. So, I wrote a book to help them. As a result of that first book, I was given the opportunity to do academic research into what happens behind closed doors when parents help their children with mathematics and ended up with my PhD from the University of Bristol. Since then I have been putting these ideas into practice, writing books to help parents, working with parents in school, and running courses and giving talks in out-of-school settings.

Wright's Background

Having trained as a primary school teacher, I have over 20 years' experience in school improvement with a specialism in mathematics. Over the years, I have worked with a range of staff, pupils and parents, focusing on improving mathematics teaching and learning and developing a love of mathematics through my role in the local authority and as a co-director of an education consultancy business. With the introduction of the National Centre for Excellence in the Teaching

of Mathematics in 2006, I took on a part-time role as a regional co-ordinator for a few years alongside my advisory work in the local authority. This relationship, and the inspiration of being part of a community of like-minded professionals, led to current day partnerships with my local mathematics hub. Through the hub, I joined a work group led by Sue Johnston-Wilder focusing on helping young people overcome mathematics anxiety and this resulted in my interest in supporting parents and carers in helping young people at home.

What are the key messages regarding Parental Engagement and Developing Mathematical Resilience that parents need to know?

Developing Mathematical Resilience is about developing and maintaining a positive stance, a 'can-do' attitude towards mathematics. What are these key messages that parents need to know? To answer this, we shall be looking at what we know from the literature under three headings:

Parental Engagement

Parental Engagement and Mathematics

Developing Mathematical Resilience

Parental Engagement

Parental engagement is to be welcomed as a very good thing in general (Desforges & Abouchaar, 2003) and there is a great deal of literature to support this. But what is it that parents do that is important? Harris and Goodall (2007) found it is not academic ability, but what the parent does with their child that has the most impact. It is engagement that really helps. Taking an interest in their child, having conversations.

However, if parents with mathematics anxiety helped their children at home with mathematics, Maloney et al. (2015) found this can have a negative impact on progress. However, Goodall, Johnston-Wilder and Russell (2017) report that parental mathematics anxiety can be overcome, leading to a positive experience for parent and child.

Parental Engagement and Mathematics

When considering parental engagement and mathematics, it is important for parents to listen to their child and not ignore or dismiss their child's answers but instead to get to know their child's thinking. One way to do this is to ask questions such as, 'How did you get this answer?' In summary, start from where they are (Hughes, 1986). Parents need to be aware that there if often more than one valid way to calculate an answer. I found an instance where the parent said school was teaching subtraction in the wrong way as the parent did not recognise the method of subtraction the child was using. The school was teaching subtraction by decomposition, and the parent had been taught subtraction by equal addition. They were unaware there is more than one valid way to calculate a subtraction (Russell, 2002). It is also important to pass on a positive attitude and not speak negatively about mathematics (Eccles & Jacobs, 1986).

Developing Mathematical Resilience

Sharing the Growth Zone Model, developed by Lee and Johnston-Wilder (2017) which will be discussed below, with parents and children can be very helpful (Maths on Toast, 2019). It can help the child express their feelings as they are learning mathematics and parents to understand what their child is feeling and so empathise and give support.

The Growth Zone Model helps learners understand and express how they are feeling about mathematics. It also helps the parent to know what their child is feeling.

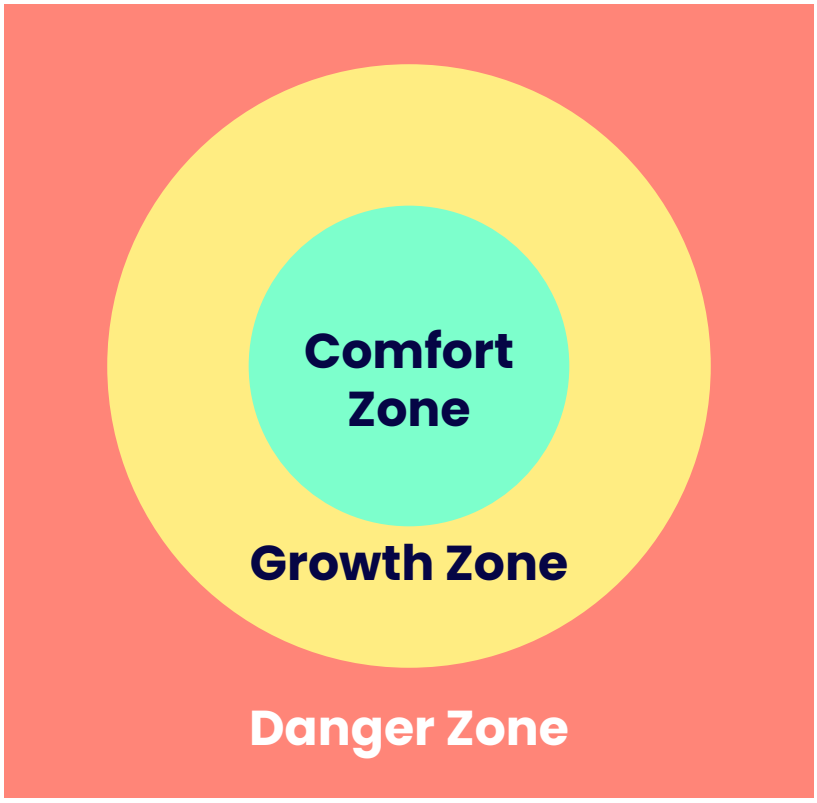

Figure 1: The Growth Zone Model

Referring to the diagram, the Comfort Zone (green) represents where the child feels safe. They say they are in this zone when they can answer questions on a topic without help. The Growth Zone (amber zone) represents the feelings of a learner when new work is introduced. The child is learning new ideas, making new connections. It can be very exciting but also challenging and therefore can feel risky. The Danger Zone (red zone) represents a time when in the course of learning, the child may come across a topic that causes them to feel anxious or to panic. They have lost the all-important link to their secure knowledge. There are many reasons why this can happen, such as perhaps the teacher moved on too fast or the child missed something due to absence. It is important to realise what has happened and find a positive way out of the Red Zone.

The intention in developing mathematical resilience is to enable learners to stay longer in their growth zone. In order to help someone develop mathematical resilience, there are four areas that need to be considered: growth mindset, valuing mathematics, understanding of how to work at mathematics, and knowing how to recruit help.

Growth Mindset

Firstly, having a Growth Mindset. This is based on the work of Dweck (2000). Ability is not fixed; it is not a case of either you can or cannot do mathematics. Everyone can improve. A failure or mistake should be considered an opportunity to learn. The adage, 'It is not that you can't do it, it's that you can't do it yet,' is very helpful.

Valuing Mathematics

Secondly, the value of mathematics should be highlighted. It is part of our lives, from a toddler learning which shoe goes on the left and which on the right foot (Spatial Reasoning), to making budgets. It is an important part of society, for instance, it is needed for design and engineering. Also the learner's own mathematical contribution is of value, when a child can use mathematics they can understand ideas and help others.

Understanding how to work at mathematics

Thirdly, an understanding of how to work at mathematics is important. At times, a student may struggle to grasp concepts. They will need both perseverance and persistence to overcome barriers. Mathematics also has structure; learners build on previous work and need to fit in new learning to what they know. (Skemp, 1971), and so it is important that each stage is understood each stage. There is also no fixed way of doing things, learners may need to experiment. And in all of this, students need to manage their emotions. The Growth Zone Model helps express how learners feel. Being in the Red Zone is not pleasant. They may know many things, but it can all seem so far away. Learners need to find a positive way out of the Red Zone.

Knowing how to recruit help

Fourthly, an understanding of how to recruit help when needed. This is particularly helpful if a student finds themselves in the Red Zone. There are many strategies available, such as working in collaboration. Techniques such as deep breathing may also help.

How can these messages be put over in a meaningful and accessible way?

We next consider how all of these messages can be put over in a meaningful and accessible way. Some parents have practical difficulties and face child care issues or are very busy so find attending meetings at school difficult (Harris & Goodall, 2007). Others may have negative attitudes to school (Skyrme, Gay and Ratcheva, 2014). In order to overcome this, I have successfully run courses in homes, written books and articles giving support to parents (Russell, 2002).

In 2018, Dorset Library Services approached me to give an author talk as part of their Libraries Week celebration of local authors. So, I gave an author talk to parents about how to help give your child a 'can-do' attitude to mathematics (Russell, 2018). Other talks followed. After one of my talks, a parent contacted me saying how helpful she found the talk. She put into practice what I said, and her daughter's mathematics scores doubled. I was encouraged to continue giving these informal talks (Russell, 2019).

New ways of communicating ideas about mathematical resilience to parents

Due to the Covid-19 pandemic restrictions, library talks, as public gatherings, were disallowed. However, this opened up new ways to reach parents. Firstly, in 2020, I reworked the book I was writing, and the educational turmoil of lockdown made me realise the importance of the topic I was addressing (Russell and Wright, 2022). The new book, Help Your Chid Do Maths Even If You Don't (2020), focuses on key messages about parental engagement and mathematical resilience.

Secondly, my recent author talks, based on my latest book have been given online. These online author talks led to me being introduced to Donna Wright who, through her role as a Senior Education Improvement Adviser for Solihull, had developed ways of offering support to parents who had found supporting mathematics very challenging. Through Wright I worked with NCETM Origin Hub, giving my author talk to school leaders in Coventry, Warwick, and Solihull. They wanted to hear the messages I was giving and how I communicated these messages with parents. Invitations for attending my online author talk were then sent out to parents from these schools. I gave my online author talk and Wright co-hosted the meeting. Those attending were able to ask questions and participate using the Chat facility and also (at the end) by unmuting their microphone.

The event was oversubscribed, and the talk was well received. Many contributed through the Chat facility. The content of the talk was helpful to parents and there were many appreciative comments. Here are some quotes taken from the Chat:

> *"I just love the 'I can't do it yet' attitude instead of 'I can't do it.' Can apply it to my professional life too. Thank you so much for that."*

> *"I definitely have issues with my own mathematics and am mindful I don't want to pass this onto my child. The points about how to get out of the red zone make total sense and have made me now think how I can deal with problems and make it better for me and my child."*

> *"I think it's untraining my brain in how I was taught and learning to think about mathematics in a different way, a more positive way. I would also be interested is this talk given to schools and teachers?"*

Way forward

Some of the parents who missed the event have contacted Wright, requesting that the talk be run again. As a result of these encouraging requests, together with the very promising results from the Chat feedback, further author talks are being planned.

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Paper 10

Towards understanding the development of mathematics anxiety – the mathematics life story framework

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Abstract

Various research methods have been used to facilitate the study of mathematics anxiety (MA) including measurement scales and autobiographical methods. Interpretation of scores from MA scales can be problematic, as there is no gauge for what score constitutes a high level of MA, nor does the score uncover the etiology of the individual's level of MA. In this regard, the scores of MA scales can be elucidated when combined with qualitative methods to allow greater insight into likely sources of MA. This paper documents the trial of a new research method – the mathematics life story (MLS) framework – to facilitate interpretation of mature students' MA scores. To test the framework, summaries of two mature students' MLSs are presented. The findings show that the MLS framework provides a structured and comprehensive method of exploring the adult learner's life experiences with mathematics, thereby helping to identify what contributed to their denoted level of MA.

Keywords: mathematics anxiety, mature student, mathematics life story framework, service mathematics.

Introduction

Since the 1960s different research methods have been employed to study mathematics anxiety (MA) (Dowker et al., 2016); for example, measurement scales have been used to gauge the extent of MA (Cipora et al., 2019); and variations of mathematics stories (Coben & Thumpston, 1995) have provided space for individuals – and especially adult learners – to convey how MA has impacted them. While many measurement scales use Likert-type formats to denote levels of MA in different contexts (Richardson & Suinn, 1972), the outcome is a total score for each respondent. Interpretation of scores can be problematic, as there is no gauge for what score constitutes a high level of MA (Carey et al., 2019; Dowker et al., 2016). Further, examining individual responses to the measurement scale does not uncover the background to the individual's level of MA. However, the scores of MA scales can be elucidated when combined with qualitative methods to allow greater insight into likely sources of MA (Drake, 2006).

This paper explores the development of a mathematics life story (MLS) framework as a method of facilitating interpretation of adult learners' levels of MA through their mathematics life story. The framework comprises a set of themes or questions that help to tease out an individual's experiences with mathematics throughout their lives. It is a qualitative method to help students uncover incidents where individuals have encountered mathematics and to examine the significance of those incidents in their engagement with mathematics.

Background

Many people like sharing stories about their time at school and particularly if those episodes were memorable, inspirational or difficult (McAdams, 1993). We are practitioners working in higher education and teaching service mathematics, that is teaching mathematics to student whose main course content uses mathematics. Over the years we have been approached by many students who have shared their mathematics stories; some would immediately express their fear about doing service mathematics and would talk about their difficulties doing school mathematics, or how they had a bad experience that changed the way they viewed and approached mathematics. This was the nature of many of the stories we heard, and they prompted further investigations into MA and the stories behind mature students' feelings about mathematics (Ryan, 2019). While much research on MA uses quantitative methods, insights into the 'when and why' (Cohen et al., 2011) using qualitative methods are central to understanding where the students were coming from.

Many studies on MA use measurement scales to determine quantitative measures of MA (Cipora et al., 2019), for example, the Mathematics Anxiety Scale U.K. (MAS-U.K.) or equivalent MA tests. Their purpose is to show a numerical measure or score for how anxious the research participant is about mathematics. MA scales typically comprise a list of statements relating to different types of engagement with mathematics in various contexts including academic and everyday situations. To complete the MAS-U.K., the respondent gives a rating of one to five to each of 23 statements, where a rating of 1 means "not at all" anxious and a 5 is "very much" anxious (Hunt et al., 2011). After all statements have been rated, the sum of the ratings gives an overall measure of MA. MAS-U.K. scores can range from 23 (the lowest level of MA) up to 115 (the highest level of MA). Typically, the scores can be interpreted as a low score meaning a low level of maths anxiety, with a high score denoting a high level of MA (Hunt et al., 2011).

By examining the responses to the individual MAS-U.K. statements the researcher can gain some insight into different situations that might lead to particularly higher levels of MA. However, MA scores do not give much insight into the incidents or experiences that led to the individual's level of MA, nor are they meant to. That is not their purpose. Consequently, this disparity necessitates further investigation into the individual's engagement with mathematics so that the voice of the research participant can be included (Cohen et al., 2011) to help elucidate what might contribute to their MA (Ramirez et al., 2019).

With this backdrop, the authors identified the need to investigate MA using a mixed methods research design, starting with a quantitative phase to measure students' levels of MA, and followed by a qualitative phase using mathematics stories to elucidate the MA scores. The remainder of the paper documents the development and trial of the MLS framework as a qualitative research method to explore the life experiences with mathematics of adult learners with low and high levels of MA.

Mathematics Life Story Research

The intention for using mathematics life story research is to foster an understanding of significant events in a person's life that have involved engagement with mathematics which may be associated with their level of mathematics anxiety. Significant events come to mind very quickly and vividly for the person, particularly negative events (Newby-Clark & Ross, 2003). Some people who share aspects of their life story comment on how much of a relief it is to have been able to share their story; in this regard, there is a therapeutic element to the life story method. Further, the speaker may feel very happy to have been given an opportunity to share their story (McAdams, 1993). For some individuals, their engagement and past experiences with mathematics may have resulted in sadness, hurt, and depending on the severity of the experience they may be unable to share their stories (McAdams, 1993). However, the life story method promotes a very liberal way of describing experiences, giving the research participant great flexibility in how they convey their story.

Formulation of the Mathematics Life Story Framework

McAdams' (1993) life story framework served as a guide for the identification of periods in one's life where a student might engage with mathematics. The purpose of formulating the MLS framework is to identify those periods in a student's life where they would have engaged with mathematics and to allow the student to explore what that engagement was like using different themes to guide them through the process of looking back at their mathematics life story. The research participants in the study were mature students aged 23 and above. Taking their ages and life stages into account, seven themes were identified as relevant to finding out about their engagement with mathematics:

1. Primary school context
2. Secondary school context
3. After school/the work context
4. Current programme of study
5. Future career
6. Significant persons (e.g., teachers, parents)
7. Strategies with mathematics.

Themes 1, 2 and 4 relate to experiences with mathematics in the school or higher education context, while theme 3 explores life and work experiences that involve engagement with mathematics or numbers. Theme 5 asks participants to consider how mathematics might feature in their future lives or careers. Theme 6 focuses on significant people in the mathematics journey, particularly teachers, and parents, as these feature in the literature on antecedents of MA (Cipora et al., 2019). Since all students being interviewed were involved in higher education at the time of interview, it was important to find out about their strategies for learning mathematics (theme 7). The intention was to ascertain how they had succeeded with mathematics up to that point. They would need to have had a certain standard of mathematics for entry to higher education and to study the degree course which the mathematics as a service subject supported. These seven themes formed the basis for the questions used in the interview setting with the mature student participants.

Findings

Two mature students were interviewed using the MLS framework; Ken, aged 46 with low MA (MAS-U.K. score 31) and Pat, aged 34 with a higher level of MA (MAS-U.K. score 94), and the responses of both interviewees were compared. The following paragraphs are extracts from Ken's and Pat's responses to the MLS framework questions on the seven themes listed above.

Primary school context

Ken: *"Everything with maths was based on you knowing your times tables, … and I used to get punished in school if I didn't know them."*

Pat: *'It's been stressful since I started primary school. … If I got individually asked I'd blank…. I've never been good at maths, never, never, … back to times tables."*

Secondary school context

Ken: *"It seemed all [teacher] wanted to do was algebra, like for two years we did nothing but algebra. And I just got sick of it in the end, and I passed the exam, but to this day I don't know how."*

Pat: *"Algebra was mystery to me. How could you get a letter; there's a number… And you don't really use it in day-to-day, like you wouldn't use it."*

After school/Work context

Ken: *"I'd be like measuring distances and subtracting distances and things like that. There was nothing complicated. I wouldn't say remedial maths, but you know a very basic maths."*

Pat: *"I worked as a spray painter, panel beater and there was a bit of maths involved, of adding and subtracting, mixing paint and you had to take small amounts of paint and add them up and I was able to do it no problem, once I was shown the steps."*

Current programme of study

Ken: *"It's like you get taught to write English, and then you go into an exam and it's written in Spanish. It's like where do I start you know, a lot of the times once I start the problems I find I can actually do them."*

Pat: *"Loads of numbers going up on the board and you wonder where did he get that number from, but you don't want to ask, and that goes through my head every class, it's been like that always. I need to go step by step, but they go really quick through it."*

Future career

Ken: *"I don't think I'll ever be totally comfortable with it. It will always frustrate me, and I'll always be wary of certain aspects of it, because I don't understand it."*

Pat: *"If I don't come up against it, it won't bother me. … But I'm going to [work] in hospitality, and [maths] will be in everything that I do, and I just have to get on with it. That's the way I look at it, I don't like it, but I have to get on with it."*

Significant person - Teacher

Ken: *"He would ask someone a question, and if they got it wrong, he would be like oh 'are you a moron?' you know or words to that effect. And I got to the point where I wouldn't even go into the classroom."*

Pat: *"When the teacher asked me questions and I didn't have a clue what the answer was, I'd count on my fingers under the table and that doesn't work, so it took me ages to work something out. It would have taken the class seconds, but it took me minutes."*

Strategies with Mathematics

Ken: *"I have to keep going back to the beginning and then work my way forward. Eventually it will sink in. … I have an awful habit of missing out on the smaller things or forgetting them. … I've learned to actually take more notice of that. And it has made a difference, as in it has cut the frustration level down."*

Pat: *"I know I just need to get 40, the minimum and whatever beyond that is a bonus. I don't mind, once I pass. … Take my time, think it through … think what they're asking you."*

Discussion

While it is beyond the remit of this paper to present the entire transcripts of the two interviews, the findings documented above present a brief insight into the mathematics life stories of two mature students, Ken and Pat, with low and high levels of MA respectively. The contrast between the two students helps to elucidate the differences between low and high mathematics anxious students. In particular, Ken is proactive about learning and keen to understand and see relevance in mathematics. He wants to do well in exams and will seek support when needed. He has a good self-concept in mathematics, despite having experienced intimidation in school mathematics, and he has shown resilience in his engagement with his undergraduate mathematics coursework. He is comfortable using calculations or numbers in everyday contexts.

In contrast, Pat is highly anxious about mathematics and has trouble grasping concepts. He is more likely to rote learn and follow procedures rather than trying to understand. His aim with mathematics is to focus on just passing an examination. He has poor self-concept in mathematics and is less likely to seek support in order to save face. Pat experienced intimidation and failure in school mathematics, and has difficulty moving on from those experiences. He has had difficulty actively seeking the support of teachers. His preference is to avoid engaging with calculations or numbers in everyday contexts, although he has demonstrated resilience in gaining a place in his undergraduate program of study. He also accepts that he will likely encounter mathematics in his future career, but he is prepared to take them on board.

In both interviews the students' shared details of many experiences with mathematics including some experiences that were not represented by the above themes. Consequently, there is potential to develop the framework further to include additional themes relevant to the mature student's engagement with mathematics, including consideration of getting support with mathematics, and more focus on their engagement with mathematics in higher education.

The experience of using the MLS framework for these interviews has been very positive. The framework provides the interviewer with a set of themes or questions to capture the mathematics story of the research participants. Thus, the framework gives structure to the interview, providing themes for comparison, while handing over responsibility to the interviewee to share their stories. Using the two research methods sequentially – MAS-U.K. test followed by the mathematics life story interview – worked well for this study. The MLS framework helped to elucidate the stories behind the MAS-U.K. scores. The findings allowed comparison of issues pertaining to mature students with low and high levels of MA and contribute to a better understanding of the factors that lead to different levels of MA.

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Paper 11

Can collaboration between Mathematics Teachers and Student Engagement Coaches impact learner engagement and motivation? A study undertaken at Cambridge Regional College.

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Abstract

The aim of this project was to determine whether employing Student Engagement Coaches (SECs) to work in partnership with Mathematics Teachers could help improve attendance, motivation and engagement for students on a post-16 GCSE mathematics resit course. Using evidence-based approaches, SECs and teachers planned and delivered lessons to whole groups together to ensure students barriers to learning were addressed. In addition, students who were identified as being particularly mathematics anxious and/or avoidant were offered 1-1 coaching sessions. Qualitative and quantitative data was collected in the form of pre- and post-intervention questionnaires and staff focus group discussions. Qualitative data was analysed using thematic analysis. Post-intervention data indicated that student confidence and engagement had improved, 84% of students stated that working with a coach improved how they felt about their mathematics ability. Other intervention outcomes included an increase in student agency and resilience.

Keywords: Student engagement; mathematics anxiety.

Background

In 2017, the Department for Education commissioned the Education and Training Foundation to set up an educational project to deliver sustained improvements in mathematics for sixteen-to-nineteen-year-olds, in post-16 settings. The program involved setting up Centres for Excellence in mathematics in twenty-one different post 16 settings. As noted by Ofsted (2021):

> *In the English mathematics education system, emphases on reactive approaches are associated with a wide attainment spread and a long tail of under-achievement. Almost 180,000 students had to re-sit GCSE mathematics in 2019. Of these, only 22.3% achieved a standard pass (grade 4) or above. (Ofsted, 2021)*

Further Education (FE) Colleges generally offer courses in vocational subjects such as Construction, Hair and Beauty and Uniformed Public Services. The students tend to be more practical and perhaps have not succeeded in traditional 'academic' subjects at school and therefore FE students are more likely to be re-taking GCSE Mathematics and English than those that attend sixth-form colleges. FE colleges take students with generally lower-than-average mathematics grades. It is common for students to enter a mathematics resit cycle at college where having not achieving a grade 4 at school, they go on to re-sit GCSE mathematics again and again over the next three years, only stopping when they get a grade 4 or leave college.

The student demographic of FE colleges is such that a high proportion of students have an Educational Health Care Plan and/or have had a negative experience of school. This means they often arrive at college de-motivated, disengaged and mathematics anxious.

Cambridge Regional College (CRC) offers vocational courses for school leavers, professional training, qualifications, and community courses including English and mathematics. As one of the Centres for Excellence in Mathematics (CfEM), the college has had the opportunity to explore, through action research, ways of improving student motivation, engagement and ultimately achievement. The implementation of the CfEM gave the opportunity to run practitioner action research into improving student outcomes at GCSE re-sit. The CfEM at CRC made the decision to employ separate people to mathematics teachers to aid student motivation, engagement and build mathematical resilience in ways that mathematics teachers did not have the time to do. As highlighted by Marshall & Johnston-Wilder (2017), students need guidance in overcoming mathematics anxiety:

> *"A few participants described themselves as previously "attempting to remove … mathematics anxiety but without a sort of strategic plan of how to do so". For other participants, helplessness manifested as a 'that's life' acceptance that mathematics anxiety is a problem but not knowing what to do about it." (Marshall & Johnston-Wilder, 2017, p.9).*

It was agreed that the role of these newly engaged people would centre on coaching, with a focus on helping students build mathematical resilience. Coaching within educational contexts has been recognised as a powerful tool for the academic, professional, and personal development of students, teachers, and senior leaders. Coaching outcomes have also been associated with an increase in 'cognitive hardiness', i.e., the ability to recognise opportunity for adaptation and change in the face of stressful situations (Kobasa and Maddi, 1977) – a key component of resilience (Bonanno, 2004).

Common barriers to engagement

Whilst our research findings highlighted a wide range of reasons some GCSE mathematics resit students struggle to engage with the subject, a number of common barriers were identified from the literature including:

Negative experiences of school

A number of students arrive at college having had negative experiences at school. This can include repeated failure, teacher-learner conflict or being made to feel embarrassed for not being able to perform mathematics 'correctly'. If instances of this nature have not been dealt with/rectified at the time of occurrence, these experiences seem to lead to students avoiding similar situations (mathematics classes) in an attempt to protect themselves from it happening to them again.

Previous failure

As noted by Johnston-Wilder et al. (2015), repeated failure often leads to one of two consequences, "namely increased determination, leading to improvement in performance, or emotional harm and helplessness, anxiety, inactivity, and fear, leading to reduced performance" (2015, p.2). By not achieving a grade 4 or above in mathematics, GCSE mathematics resit learners have all experienced some form of 'perceived failure'. Many of the target learners adopted a fixed mindset, believing that the process of improving their mathematics skills is outside of their control and

therefore, despite the amount of effort they put in, their ability will not allow them to achieve a grade 4.

Lack of self-efficacy

Self-efficacy is defined as an individual's belief in their ability to "organise and execute courses of action required to attain designated types of performances" (Bandura, 1986, p.391). In a mathematical context, self-efficacy can relate to a learner's belief in their ability to execute a mathematical task; a student with high levels of self-efficacy, for example, would hold the belief "I can do this mathematics problem" and would be more likely to have greater perseverance in challenging mathematical tasks than a learner with low self-efficacy (Pintrich and Schunk, 2002).

Mathematics anxiety

Mathematics anxiety is commonly described as feelings of "tension and apprehension that interferes with mathematics performance ability, the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations" (Khasawneh et al., 2021, p.1). Mathematics anxiety manifests itself in number of ways and can cause a range of physical and psychological symptoms such as sweating, nausea, increased heart rate and feelings of helplessness. Lyons and Beilock (2012) found that mathematics anxiety is a conditioned anticipatory fear of mathematics, as opposed to a reaction of doing mathematics itself. Through both primary and secondary research, mathematics anxiety has been found to have a profoundly negative impact on learner engagement at CRC and therefore requires the appropriate time, space and intervention to be minimised effectively.

Methodology

Student Engagement Coaches (SECs) were previously employed as part of the research design for CRC's 19/20 and 20/21 action research projects. In previous years, learners who were identified by their mathematics teachers as lacking motivation and/or engagement in mathematics were referred to the SEC team and were allocated a SEC on either or short-term or long-term basis, depending on the needs of the learner. The aim of this year's study was to build on our previous research findings by testing whether teacher-coach partnerships could impact the motivation and engagement levels of GCSE mathematics resit students.

Students were coached collectively as a class after being identified as having low engagement levels. Three pairs of mathematics teachers and SECs planned and implemented mathematics sessions together; mathematics teachers focused on the academic aspect of the content and the delivery, whilst the SECs approached the planning with an emphasis on the emotional and psychological needs of the students. Students who presented as particularly disengaged, demotivated or mathematics-anxious were also supported in a 1-1 coaching capacity. This allowed the SEC to gain a deeper understanding of the barriers faced by the individual and provided an opportunity to work together in a space separate to the mathematics classroom.

Coaching was delivered using a 4-step framework: observing behaviour, relationship building, intervention and reflection. The framework was created to allow SECs the time to identify learners' individual barriers to engagement and to focus on building a professional relationship based on trust, before dealing with the mathematics itself. As aforementioned, for learners who have had negative experiences with previous teachers, it can take time for new, trusting teacher-student relationships to be built – a key predictor of student engagement (Roorda et al., 2011). It was also important that SECs adopted a flexible approach to intervention, as what works for one learner may not be effective for another. Interventions were implemented

in both class and 1-1 settings and included, but were not limited to, confidence building techniques, challenging negative self-talk, encouraging a safe classroom culture, modelling help-seeking behaviour, diffusing tension and exam anxiety techniques such as graded exposure to past papers and teaching breathing techniques. Classrooms used as part of the project were also arranged in a way which encouraged engagement and a growth mindset, as well as facilitating an adult learning environment. This included the rearrangement of tables as well as whiteboards, non-mathematics related posters and plants.

Data Collection Methods

A number of methods were used to analyse the effectiveness of the intervention. Prior to the intervention, qualitative and quantitative data were collected in the form of a student questionnaire. The questionnaire was split into two parts. A series of open and closed questions were structured to discover the students' attitudes towards mathematics at the point of completion. The latter part of the questionnaire was an adapted version of The Abbreviated Mathematics Anxiety Scale (Hopko et al., 2003), where students were given a series of short hypothetical scenarios within a mathematical context and asked to rank how they would feel from 0 (extremely worried) to 10 (very comfortable). Not only did this enable teachers and coaches to determine student starting points, but it also allowed them to discover how students were feeling in certain situations, prompting them to ensure that students were not put in situations that would increase stress or anxiety. Similarly, it allowed support to be planned in advance where stressful situations could not be avoided. A post-intervention survey was also given to students to identify any differences in attitude towards mathematics, self-confidence, or self-efficacy as well as whether their exam experience had been altered by working with a coach. This survey also took the form of open and closed questions.

Staff focus groups, containing teachers and coaches involved in the action research project, were held to create an additional source of qualitative data. A series of these focus groups took place towards the end of the academic year and to ensure bias was kept to a minimum the structure and discussions were led by a consultant who was independent of the action research project. Allowing someone who was external to the project to design focus group questions and determine the direction of the discussion allowed for increased objectivity. These meetings were transcribed from audio recordings before any errors were amended and analysis took place.

Qualitative Data was processed using thematic analysis, a method of systematically identifying and interpreting patterns in the data (Clarke and Braun, 2006). To enable themes to become visible, data is sifted through carefully until it is possible to spot evolving themes. Codes were then created from this which formed the foundation for the themes. This thorough process of analysis gave a strong basis to consolidate and detect any significant patterns within the data collected.

Results and Discussion

Analysis of the pre- and post-intervention questionnaires highlighted several key findings. For example, 84% of learners said that working with a SEC improved how they felt about their mathematics ability. Linnenbrink and Pintrich (2003) found that students who believe in their ability are more likely to "be engaged in the classroom in terms of their behaviour, cognition, and motivation" (2003, p.136). Another significant finding was that 88% of respondents reported that being supported by a SEC during the exam period had a positive impact on their experience. Mathematics exams can be a significant source of stress for GCSE resit students and can trigger feelings of anxiety which may not have presented themselves in the teaching year. Therefore, the exam period was a key focal point for SECs and more intensive coaching was offered to learners during this time.

A thematic analysis of staff focused group discussions revealed 2 main themes: impact of the intervention and reasons for that impact. These were then broken down into five sub-themes: student impact, staff impact, classroom culture impact, intervention strategies and challenges.

Action research participants evidenced several constructive ways in which the intervention impacted students' attitudes and behaviour. This included an improvement in self-confidence, a reduction in the fear of making mistakes, a willingness to move out of their comfort zone and attempt more challenging questions. In addition, reduced exam anxiety was aptly summarised by a teacher:

> *"I think our students were definitely more confident going into the exam [compared to students not in the intervention]. They were not having panic attacks and meltdowns. …. some students were so distressed from other departments. On the whole, our students were much better prepared psychologically for the exam than other departments were, … Our quality manager was saying 'get me a coach, get me a coach, get me another one' because there were so many issues."*

The second theme was centred around the impact staff had on each other during the intervention. Staff in the Action Research Group reported that they had learned a lot from their colleagues, both in the classroom and when planning a lesson together. Discussion revealed that this learning was born out of mutual respect and a recognition of specific skills in the mathematics teacher and SEC role. For example, both coaches and teachers agreed that they had, through continual communication learned more about the students: the gaps in their mathematical knowledge, their anxieties, and reasons for them, as well as their personal background. This acquired knowledge helped them to attend more closely to, and develop students' engagement, and mathematical understanding.

Whilst classroom culture is interrelated and dependent on student impact and teacher impact, participants outlined a number of key features of classroom culture which developed over the course of the year. For example, participants were keen to stress how their mathematics classes had become safe environments for students without the fear of feeling 'stupid'. A culture was nurtured in which students were made to feel valued and respected, alongside the creation of an adult environment. This in turn fostered a sense of equality amongst coaches, teachers and students, and led to a reduction in anxiety levels amongst the student sample.

The importance of having a flexible approach towards the facilitation of intervention strategies was highlighted throughout the focus group discussions. For coaches, it was noted that the effectiveness of the intervention relied heavily on the cause of demotivation/ disengagement for the student and their willingness to participate. Interventions which were reported as having a significantly positive impact on student engagement were those which focused on improving emotional engagement, relationship building between staff members and learners and confidence building. It was also identified that students who presented as exam-anxious required separate, more specific interventions which gave them the space to explore their negative feelings and the tools to help manage them.

Following the Covid-19 lockdowns and its impact on students' education, student behaviour was exceptionally challenging this year and manifested itself in several unique ways. Many students were noted to be lacking in the 'soft' skills usually built in their last year/s of school. It was observed that many students found classroom relationships challenging, struggled to comprehend classroom boundaries and had a distinct shortage of autonomy. Other challenges faced by the teachers and coaches were of a more logistical nature, such as timetabling constraints and having to share classes with non-action research teachers. It was

also found that many students took a longer time to build up trust with teachers and coaches and were therefore slower to take up offers of intensive support.

All members of the action research group recognised the continual challenge of active listening, which needed them to abandon their own agenda for the mathematics session in order to be responsive to student feedback. As reflected on by a coach, if a member of staff was not responsive to student feedback then this could hamper the development of trust between them and the student and ultimately hinder student progress. It was also noted by many of the group that building trust takes time, which is challenging when there are only two or three contact hours per week. Teachers alluded to the fact that they had an internal conflict between 'we're here to teach' against allowing students the time and space to have a 'little bit of a laugh' in order to build relationships.

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Paper 12

Maths ACTive: Mastering Mathematics with Psychological Flexibility

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Abstract

Our rich understanding of *Mathematics Anxiety* (MA) has resulted from the varied repertoire of approaches in which it has been studied. However, whilst rarely stated explicitly, much of the research undertaken has taken a pathological view of MA – assuming that there are abnormal psychological processes at its root. 'Causes' and 'symptoms' are sought; 'courses' of development are tracked across time and location; and 'treatments' are devised and tested for efficacy. Coherence across all three ventures is rare. Conversely, *mathematical resilience* (MR) offers a positive and practical approach to addressing MA via the identification and promotion of the adaptive processes that allow learner to thrive in mathematics. In order to unify both of these efforts, a new conceptualisation of MA and MR is presented here that is based upon the assumption that the two constructs result from the same 'normal' psychological processes. Acceptance and Commitment Training (ACTr) delineates six such dynamic processes and behaviours, collectively known as *psychological flexibility*, and provides a functional and empirically grounded account of how to develop it. Adopting this framework potentially opens a wealth of evidence-based principles and techniques that can support learners in working *with* their anxieties to become more mathematically resilient. An online mathematics mastery course for school students, *Maths ACTive*, has been designed to determine if such an approach is indeed as fruitful as previous research suggests. This presentation will first explore the new model before explaining the process and challenges of adapting ACTr for use with secondary mathematics students in an online medium.

Keywords: Acceptance and Commitment Training; Psychological Flexibility; Mathematics Mastery; Mathematical Resilience.

'Mathematics Anxiety is not a disease'

No miracle cures have been discovered; in fact, once the disease is diagnosed, recoveries seem to be marginal at best.' (Martinez, 1987). A 'disease' is well-defined: it has a characteristic set of causes (etiology); a known course of action; and, hopefully, a distinctive response to treatment. For example, COVID-19 was a relatively straightforward viral disease – its devastation was due to behavioural, social, and political reasons. Whilst MA is rarely described as a disease, I believe that this way of thinking remains current in research and understanding of the issue.

'Assumption of healthy normality'

The field of physical health is underpinned by the notion that the human body is meant to be healthy. Infection, injury, toxicity, aging, or decline in physical capacity can then disturb this healthy normality, and result in aliments that are often discussed with medical doctors. The analysis is simple: (1) identify the syndromes (the signs and symptoms); (2) find the abnormal

processes causing them; and (3) find ways to alter these processes. This has been a remarkably effective model in advancing medicine. As society values being free from distress, these principles are naturally applied to mental health: human are inherently happy, connected with others, altruistic, and at peace with themselves *but* can be disturbed by particular emotions, thoughts, memories, historical events, or states of the brain. The undesired psychological syndromes are identified, mechanisms are proposed, and then the causes are found and resolved. The result is the widespread belief that *abnormal* processes are at the root of both physical and mental disorders. MA is not a disease, but much research is based on the (often implicit) assumption that MA is caused by abnormal psychological processes. In order words, something is not quite going the way it should for MA to arise.

Assumption of destructive normality

If the assumption that abnormal processes cause MA is rejected, what is the alternative? *Acceptance and Commitment Therapy* (ACT; Hayes et al., 2012b) conceptualises the issue differently. It asserts that psychological disorders, and human suffering more generally, predominantly involve the misapplication of otherwise 'normal' psychological processes in destructive and dysfunctional ways. ACT has identified six such processes which form its clinical goal: *psychological flexibility.* These processes involve the behaviour of 'languaging' (Hayes et al., 2001), particularly its symbolic and problem-solving features, and give rise to both human achievement and human misery. 'Abnormal' processes do still exist – a brain injury often results in uncharacteristic behaviour –however, misapplied, the six core processes can amplify and exacerbate the psychological suffering of the situation.

Psychological Flexibility

Kashdan and Rottenberg (2010, p. 865) define psychological flexibility (PF) as the ability to

> *"recognize and adapt to various situational demands; shift mindsets or behavioral repertoires when these strategies compromise personal or social functioning; maintain balance among important life domains; and be aware, open, and committed to behaviors that are congruent with deeply held values"*

This rather cumbersome definition is due to the fact that it is describing each of the six core processes, better shown in the Hexaflex model in Figure 1.

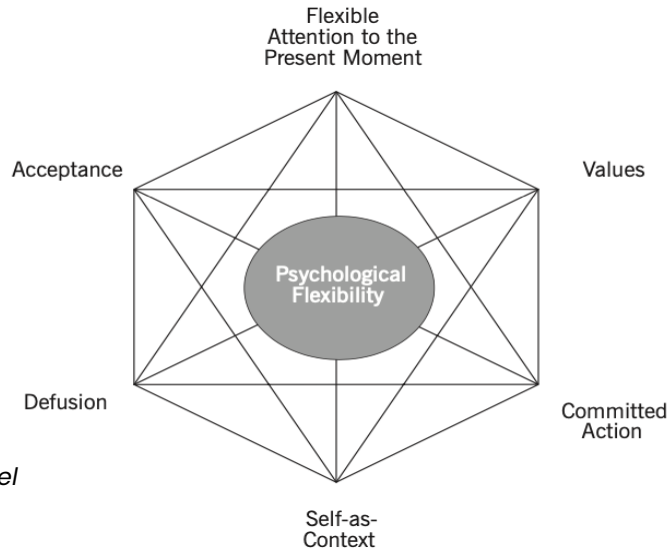

Figure 1: The hexaflex model of Psychological Flexibility. (Hayes et al., 2012b)

The resulting PF processes have been derived from several areas of research. ACT is situated within Contextual Behavioural Science – a *reticulated model* of science, meaning that the advancements made in each of its research programmes is shared with all other areas at every level in order to inform their progress also (Hayes et al., 2012a). For example, basic research on language and cognition informs clinical practice, which then feeds back into further theoretical developments. Interaction between researchers in evolutionary science, behaviour analysis, philosophy of science, and prosociality create a network of harmonious effort (Zettle et al., 2015). Whilst the psychological flexibility model is supported by a vast quantity of empirical data (Gloster et al., 2020), many of the PF behaviours are also consistent with time-old wisdom traditions and religious practices (Hayes, 1984). It has been demonstrated that practitioners focusing on a subset of the processes, or even one, can make a marked improvements to clients (Villatte et al., 2016), however, considering all six results together results in the strongest outcomes. Distinguishing the six intimately interacting components may be practically and analytically helpful, but the targeted outcome is always the whole: psychological flexibility.

Psychological Inflexibility

Before exploring the positive construct of psychological flexibility, it is valuable to explore its absence**:** *psychological inflexibility (PI).* ACT posits that the six behaviours in Figure 2 are the root cause of human suffering and maladaptive functioning.

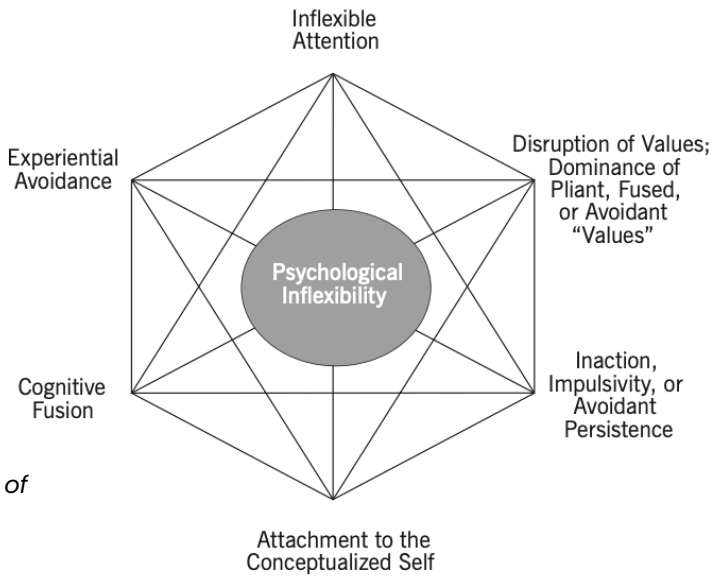

Figure 2: The hexaflex model of Psychological Inflexibility. (Hayes et al., 2012b)

My core proposal is that *Mathematics Anxiety is psychological inflexibility in a mathematics context.* I hypothesise this based on the fact that the PI model has been shown to be capable of explaining all areas of psychological suffering to which it has been applied (Hayes, 2019), and also that my initial analysis of MA as PI has proven to be equally fruitful.

Skipping maths lesson; spending hours on social media instead of revising; not putting their hand up in case they get laughed at… **Experiential avoidance** is the tendency to try to avoid or get rid of internal, unwanted thoughts and feelings by attempting to change their frequency, intensity or content (Hayes et al., 1996). Many studies have shown that this is futile (Akbari et al., 2022), and that suppression and avoidance may work in the short term, but the thoughts and feeling often come back – stronger and more often (Wegner et al., 1987). Time and energy is spent 'pushing the ball beneath the water' – but as soon as it is let go, it bounces back up again. As more and more things are avoided, life becomes more and more restricted – the anxiety prevents someone from doing what they care about. The decision seems to become doing mathematics *or* feeling anxious, rather than doing mathematics *and* feeling anxious - being willing to take the anxiety alongside.

Consider thoughts such as *'I just can't do maths'; 'I'll never be able to solve this'; 'I'm going to fail that exam'; 'I'm stupid'*. Given how often thoughts help people understand and navigate the world, it is not surprising that many can often fall into the trap of automatically believing their instantaneous thoughts. **Cognitive fusion** is about taking thoughts literally – not being able to distinguish between thoughts and what they are referring to; getting 'hooked' by them; becoming over-attached to them; and starting to behave as if they are 'true'.

Consider thoughts such as *'I don't understand a word he's saying, it's just like last week when he did the same thing, and I didn't understand it at all. It's always been the same since primary school. And I bet it will be on the test next week. I'm going to have to do so much work to catch up. I bet I'll still fail.'* In a situation that is perceived to be 'dangerous' – for some, simply being in a maths lesson – minds can start ruminating on the past or worrying about what might happen in the future. **Inflexible attention** and a rigid **conceptualised self,** make it difficult to see and make decisions in the present reality - the mind is everywhere apart from where it is needed most: right here, right now.

Consider thoughts such as *'Why am I doing this?'; 'When am I ever going to need this?'; 'I hope I get into sixth form'; 'If I don't get a good grade, mum is going to be so mad'.* It can be quite difficult for young people to articulate their **values**. If someone does not know what they care about, who matter to them, and what kind of life they want to live, doing mathematics becomes solely an act of compliance or a way of avoiding disappointing others. If someone does not know what they care about, how do they know what to do next? When they are faced with a choice at a point in life, how can they choose the most meaningful direction? Many students can become **inactive**, passive, apathetic, withdrawn, and insensitive to meaningful activity. Mathematics just happens around them.

Reports of these kinds of behaviours are a common place in the mathematics anxiety literature, as well as in many educators' experiences. This new conceptualisation of mathematics anxiety as psychological inflexibility may allow practitioners to access to the wealth of evidence-based approaches and strategies that ACT has developed and help us to support learners in pivoting these processes to upward trajectories.

Maths ACTive

My corresponding proposal is that *'Mathematical Resilience is psychological flexibility in a mathematics context'*. This proposal has been realised through an interactive online Mathematics Mastery course that I have designed: *Maths ACTive.* Over a fortnight, students are taken through a series of psychoeducational lessons, experiential exercises, metaphors, case studies, homework and quizzes to help them understand and practise psychological flexibility skills. In a non-clinical setting such as this, ACT is considered *Acceptance and Commitment Training* (ACTr). The course is based on a new definition of Mathematics Mastery: *'to use mathematics to do what matters to you',* and a novel model of mastery learning consisting of two parts: *Mathematical Understanding* – the cognitive skills needed to learn mathematics; and *Mathematical Resilience* – the affective skills needed to learn mathematics.

Instead of avoiding unpleasant thought and feelings, **acceptance** involves realising that one may not have the control to change one's thoughts and feelings as one might think. It is not about gritting teeth and getting on with it, but, learning how to turn off the 'struggle switch' when fighting does not work (Harris, 2008). Students explore the story of the huge, scary monster, *Badsadmad*, to understand this. Rather than engaging him in the constant tug-of-war to pull each other into the bottomless pit between them, students learn that there is an alternative. They can simply drop the rope. They can choose not to spend their time and energy fighting and missing out on the things they care about – this is a powerful metaphor.

Unhelpful thoughts like *'I can't do maths'* can be listened to rather than automatically taken literally. There are many **defusion** techniques that students can learn to do this – most of which make them feel a little silly in order to create distance between the individual and their thoughts. They can say to themselves: *'I am having the thought that* 'I can't do maths''; or *'I notice that I'm having the thought that* 'I can't do maths''*;* or sing it; or say it in voice of Mickey Mouse! Distance can also be created when students write down their 'sticky' thoughts and feelings on sticky-notes and physically move them around. They can 'carry them around' and still do what matters to them, without needing to get rid of them.

A great deal of the course focusses on **values**-work. The exercises encourage the students to 'think big' in order to identify what matters to them. A powerful entry into values-work is thinking about *who* matters to them, and *how* they would act toward these people. The resulting qualities of action are then their values. Work is done to help students distinguish between values and goals. Values tell people who and what matters to them - *the direction they want to move;* Goals are actions that can be completed– *what to do to move in that direction*. Finally, values are used to plan for **committed actions** – what can be done (in the short, medium and long term) to move in the direction of values. Students consider how to make goals in mathematics SMART (*Specific, Meaningful, Allowing anxiety, Realistic, Trackable*) and are reminded to listen to their values when mathematics get difficult. Afterall, true 'success' is about living by one's values – whether or not 'external success' is achieved.

The problem with Problem Solving

As I continue to develop, test and trial the course, I am also wrestling with what I consider the fundamental challenge in mathematics education. Many consider problems solving to be a key goal of mathematics education (Drury, 2018) – some define 'mathematics' as simply 'problem solving'. It is a potent tool that mathematics educators can help students sharpen and refine. However, it is this very same sharp tool that, when directed inwards, can be harmful. Overextended into the psychological domain, the efforts of problem-solving thoughts, feelings and sensations can be futile, yet relentlessly persisted with until unhelpful anxieties develop. How can a teacher help their students distinguish between *problem-solving the mathematics* and *problem-solving themselves*? How can they learn to discriminate when the mind is useful and when it is not? To do this, the contexts that feed literality and the problem-solving mode of mind need to be identified and changed. I believe that ACTr provides educators with a rich and powerful approach to achieving this.

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Paper 13

The Mediating Role of Psychological Resilience in The Relationship Between Early Childhood Experiences and Mathematics Anxiety

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Abstract

Childhood traumas may have negative effects on psychological well-being and psychological resilience interfering with individuals' ability to cope with change, difficulty or distress. One of the causes of mathematics anxiety may be low psychological resilience. The mediating role of psychological resiliency in the relationship between childhood traumas and mathematics anxiety is examined. Data were collected from 382 ninth grade Turkish students using the Psychological Resiliency Scale SV, Childhood Trauma Questionnaire (CTQ-33), Mathematics Anxiety Rating Scale (MARS) and the demographic form. The results reveal that childhood traumas have a significant, negative relationship with psychological resilience. In addition, psychological resilience has a significant, negative relationship with mathematics anxiety in the presence of childhood traumas.

Keywords: Childhood traumas; psychological resiliency; mathematics anxiety.

Introduction

Childhood traumas which contain sexual, physical and emotional neglect and abuse have direct links with many psychological disorders. These disorders may prevent the individuals from functioning effectively in certain fields of life. Those who faced childhood traumas may have a low level of psychological resiliency and may be vulnerable to emotional and behavioural problems. One of these problems may be anxiety disorder and specifically mathematics anxiety. Psychological resilience may have a mediating role in the relationship between childhood traumas and mathematics anxiety. In this study, this relationship will be investigated.

Review of Literature

Childhood Traumas

The WHO (World Health Organization, 2016) defined childhood trauma as: individuals under 18 being exposed to neglect and abuse, which may be physical, emotional or sexual. Aktaş (2019) stated that such exposure has potential effects on children's development and mental health. Research by UNICEF (2010) indicated that 51% of children in Turkey between the age of 7 and 18 years were exposed to emotional and physical abuse; 43% physical neglect; 25% emotional neglect, and 3% sexual abuse. These findings show a high prevalence rate of childhood traumas in Turkey.

Various studies have reported childhood traumas having significant relationships with actual or future psychological disorders such as anxiety and mood disorder (Arslan & Balkıs, 2015; Gluckman, Hanson and Beedle, 2007).

To the researcher's best knowledge, the relationship between childhood traumas and mathematics anxiety has not yet been studied. An indirect relationship may be explained using the neurophysiology of stress and trauma. When trauma and stress are experienced, low levels of neural activation are detected in the regions of the brain responsible for learning, generating solutions to problems and memory, namely PFC and amigdala and hipokampus (Kenya and Vuyiya, 2020). Psychological stress is a consequence of childhood trauma, and the effects of the stress on the brain may result in limited mathematical learning.

Psychological Resilience

Psychological resilience is defined as the ability of individuals to cope successfully with significant change, difficulty or distress (Rutter, 1999). It may be explained as the interaction of protective and risk factors. Significant relationships were found between resilience and psychological disorders such as anxiety, depression and stress (Morete, Solano, Boff, Filho & Ashmawi, 2019). Psychological resilience is also thought to decrease mental health problems such as depression and anxiety among children and adolescents (Dray et. al., 2017).

In addition, many studies discuss the effects of childhood trauma on psychological resilience of individuals (Bindal, 2018; Savi-Çakar, 2018). For example Savi-Çakar (2018) found that childhood traumatic experiences may decrease children's resilience.

Mathematics Anxiety

Mathematics anxiety (MA) is defined as negative emotional and cognitive responses to mathematics or numbers associated with feelings of tension and anxiety (Hembree, 1990). These negative feelings block the ability to manipulate numbers and solve mathematical problems. A comprehensive review of the potential causes of mathematics anxiety by Carey et.al. (2019) considered these causes under three categories: environmental (e.g teacher attitudes), personal (e.g. low self-esteem) and intellectual factors (e.g. dyscalculia).

Therefore, environmental and personal factors in childhood trauma may be significant predictors of mathematics anxiety. In this research, the mediator role of psychological resilience in the relationship between childhood traumas and mathematics anxiety is investigated.

Method

Participants

The participants in this study are ninth grade high school students from ten high schools located in Izmir, Turkey. After gaining official permission from the National Education Directorate, the school counsellors in these schools were informed about the research and data collection procedures. The scales and demographic forms were transferred to an online platform and a link was generated and distributed to allow students to participate in the research. Table 1 contains demographic information about participants.

Table 1

Distribution of the Participants by Gender and Age

		N	%
Gender	Male	205	53,7
	Female	177	46,3
	Total	382	100
Age	13	4	1
	14	128	33,5
	15	209	54,7
	16	41	10,7
	Total	382	100

Data Collection Instruments

Three scales (Psychological Resiliency Scale SV, Expanded Version of The Turkish Childhood Trauma Questionnaire (CTQ-33), Mathematics Anxiety Rating Scale (MARS) and a demographic form including items regarding with age, gender, school type etc. of the participants were used to collect data. The Brief Resilience Scale (BRS), was developed by Smith et al. (2008) to measure individual resilience. The scale was adapted to Turkish culture by Dogan (2015). The BRS is a 6-item self-report assessment with a 5-point, Likert-type scale. Response options are strongly disagree, disagree, neutral, agree, and strongly agree.

Childhood Trauma Questionnaire was developed by Bernstein et al. (1994). It is a self-report questionnaire consists of 5 subscales (physical abuse, sexual abuse, emotional abuse, physical neglect and emotional neglect) with a total of 28 items. The inventory is a 5-point Likert scale, the participants indicate how often they experienced each statement (1= never true, 5= very often true). Expanded Version of The Turkish Childhood Trauma Questionnaire (CTQ-33) was developed by Şar, et al. (2021) and the number of items increased to 33 adding a new factor, over-control.

Mathematics Anxiety Rating Scale (MARS), (Suinn and Winston, 2003) is a 30-item self-report scale. MARS-SV was derived from the 98-item Mathematics Anxiety Rating Scale (MARS). The Mathematics Anxiety Rating Scale-Short Version (MARS-SV) was translated into Turkish by Baloğlu (2010). The short and translated Mathematics Anxiety Rating Scale is a five-point Likert scale (from 1. not at all to 5. very much).

Statistical Analysis

In order to investigate the hypothesis of this study, the data was analysed using the SPSS program. Model testing for a mediator variable was conducted using the PROCESS macro for SPSS, SAS, and R, this procedure produces identical outcomes comparing with SEM (Hayes, 2022). When histogram, Q-Q Plot and Kolmogorov Smirnov test of normality value ($p= .053>.05$) were taken into consideration, it can be stated that the data meet the criterion for normal distribution.

Results

Prior to presentation of the results, means, standard deviations of three BRS, CTQ(33) and MARS scales are shown in Table 3.

Table 3

Mean and Standard Deviations of MARS, BRS and CT Scales

Descriptive Statistics	M	SD	N
MARS	69,61	20,60	365
BRS	17,78	5,102	365
CT	64,41	14,00	365

In order to test the model including predictor variable (childhood trauma), mediator variable (psychological resilience) and outcome variable (math anxiety), Process Analysis method was employed. Figure 1 presents this model.

Figure 1. The Model

The results of the Process Analysis applied to the CT, BRS and MARS scores are presented in Table 4.

Table 4

The results of the Process Analysis applied to the CT, BRS and MARS scores

Step	B	SE	LL –UL of 95% CI	p
1 (CT->MA	.3171	4.97	.168, .465	0.000
2 (CT->PR	-.0764	0.18	-.113,-.039	0.001
3 (PR|CT->MA)	-.1567	.195	-.1.95, -.1.18	0.000
4 (CT|PR->MA)	.1974	0.71	.0575,.3373	0.05

The results of the Process Analysis reveal that childhood trauma has a significant, negative relationship with psychological resilience. When childhood trauma increases, psychological resilience decreases. Psychological resilience has a significant, negative relationship with mathematics anxiety in the presence of childhood trauma. Lastly, childhood trauma with mediating of psychological resilience predicts mathematics anxiety significantly. The indirect effect of childhood traumas on mathematics anxiety is greater than zero.

Discussion and Conclusion

This study has found that childhood trauma significantly predicts increased mathematics anxiety probably due to decreased psychological resilience. Additionally, psychological resilience is a significant mediator in the relationship between childhood traumas and mathematics anxiety. Similarly, Yakıcı and Kandemir (2021) found that psychological resilience is a significant predictor of test anxiety and an important mediator between childhood trauma and test anxiety. This research contributed towards filling a gap in the literature. On the relationship between childhood traumas and mathematics anxiety and mediating effect of psychological resilience.

The findings of this study revealed that childhood trauma is a significant predictor of mathematics anxiety. Therefore, understanding how the role childhood trauma plays in mathematics anxiety may provide crucial clues in developing prevention or intervention efforts. Afifi and Macmillan (2011) suggest that in order to cope with traumatic experiences, psychological resilience should be instilled. The same approach may be effective in decreasing math anxiety, too.

In this study, total scores of childhood trauma were used, therefore the relationships of subtypes of traumas such as emotional neglect and sexual abuse remain unknown. Future research may focus on these questions. Since psychological resilience is the interaction of various protective and risk factors, a more detailed model of the mediator role of those factors may also be beneficial.

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